

Analysis of Noise

Part XIII

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This column is a continuation of the set (1–12) dealing with the rigorous derivation of the expressions relating the effect of instrument (and other) noise to their effects on the spectra we observe.

As we do in each column in this series-within-a-series, we take this opportunity to remind readers that we are dealing with a continuous series of columns, and so we resume our discussion by continuing our equation numbering, figure numbering, use of symbols, and so forth, as though there were no break, except that when we repeat an equation or series of equations that were derived and presented previously, we retain the original number(s) for those equation(s).

We have now gone through the analysis of two cases pretty thoroughly. It should be apparent to the reader what our approach is, and how the analysis of these situations is attacked. Hopefully, therefore, we can now go a little faster than we have been.



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In the previous column (12), we pretty much finished up our discussion of noise that was Poisson-distributed. In one sense, Poisson-distributed noise is a special case because, for example, when we analyzed the effects of noise that was constant, we did not, until it became pertinent, consider the noise to have any particular distribution. However, because Poisson noise arises naturally out of a particular noise mechanism, and it is one that occurs in several spectral regions and in conjunction with different technologies, it was appropriate to consider it as an entity unto itself. The next noise source we consider is what we originally called “scintillation noise” and which, as we noted in the previous column, is also called by several other names: source noise, flicker noise, and others. The defining characteristic of this noise is that the variability is directly proportional to the intensity of the signal. One way it can arise is through a mechanical vignetting of an optical beam. If a piece of metal were to block, say, 1% of a homogeneous beam, then the intensity of the beam would be reduced by 1% of its total intensity, so that the absolute reduction of the signal from an intense beam would be greater than for a weak beam. Another way scintillation noise can occur is if, for example, a photoresistive detector is used, then the detector current will be proportional to the detector voltage as well as the intensity of radiation impinging on it. Then, if the detector voltage varies, the change in detector current for a given voltage change will again be proportional to the optical intensity, but with a sensitivity proportional to the detector voltage. In either case, if the amount of beam blockage or the detector voltage is random, then this sensitivity change becomes a source of random variation proportional to the signal intensity.

Another characteristic of scintillation noise is that, because it represents the amount of energy in the optical beam, it can never attain a negative value. In this respect, it is similar to the Poisson distribution, which also can never attain a negative value. On the other hand, because it is a continuous distribution, it will behave the same way as the constant-noise case in regard to achieving an actual zero: any given reading can become infinitesimally close to zero, but there is zero probability of actually achieving an exact value of zero, except in the case of a complete absence of signal.

It differs from the Poisson case in two respects, however. First, the distribution of the variations is not predetermined,

but depends on the nature of the changes causing the signal variation. Secondly, the magnitude of the changes is not predetermined, but depends on the amount of variation of the cause. At an appropriate point, we will have to accommodate this by introducing a constant representing the magnitude of the variations. We will go a bit farther in characterizing the various types of noise sources we consider, and in Table IV we list, for comparison purposes, the corresponding characteristics of the three types of noise we have or are considering.

So let us begin our analysis. As we did for the analysis of shot (Poisson) noise (8), we start with equation 17, wherein we had derived the expression for variance of the transmittance without having introduced any special assumptions except that the noise was small compared to the signal, and that is where we begin our analysis here as well. For the derivation of this equation, see reference 2. So, for the case of noise proportional to the signal level, but small compared to the signal level, we have:

$$Var(\Delta T) = \left(\frac{1}{E_r}\right)^2 Var(\Delta E_s) + \left(\frac{-T}{E_r}\right)^2 Var(\Delta E_r) \quad [17]$$

In the derivation of the transmittance noise in the case of Poisson-distributed noise, at this point we noted that the variances of E_r and E_s were proportional to E_r and E_s respectively. In the current case, the corresponding relationship is that the standard deviation of the noise on E_r and E_s is proportional to E_r and E_s , with a proportionality factor, k , that is related to the magnitude of the physical cause of the noise; that is:

$$SD(\Delta E_r) = kE_r$$

$$SD(\Delta E_s) = kE_s$$

The variances of E_r and E_s , then, are proportional to $k^2E_r^2$ and $k^2E_s^2$ respectively, and substituting these values in equation 17 gives:

$$Var(\Delta T) = \left(\frac{1}{E_r}\right)^2 k^2E_s^2 + \left(\frac{-T}{E_r}\right)^2 k^2E_r^2 \quad [140]$$

$$Var(\Delta T) = \left(\frac{k^2E_s^2}{E_r^2}\right) + k^2T^2 \quad [141]$$

$$Var(\Delta T) = 2k^2T^2 \quad [142]$$

Table IV. Comparisons between noise characteristics, including the expressions for low-noise behavior.

Constant	Scintillation		
	detector noise	Shot noise	noise
Relation to signal	Independent of signal	Square root	Proportional
Continuous	Yes	No	Yes
Variance locked to signal level?	No	Yes	No
Distribution	Not predetermined	Poisson	Not predetermined
Negativity	Negative values possible	Nonnegativity constraint	Nonnegativity constraint
Probability of zero value	Zero	Finite	Zero
Expression for transmittance noise	$\sqrt{1+T^2} \frac{SD(\Delta E)}{E_r}$	$\sqrt{\frac{T+T^2}{E_r}}$	$\sqrt{2} kT$
Expression for relative absorbance: $SD(\Delta A)/A$	$\frac{SD(\Delta E) \sqrt{E_s^2 + E_r^2}}{E_s E_r \ln(E_s/E_r)}$	$\frac{1}{\ln(T)} \sqrt{\frac{1}{E_s} + \frac{1}{E_r}}$	$\frac{\sqrt{2}k}{\ln(T)}$

$$SD(\Delta T) = \sqrt{2} kT \quad [143]$$

Given the simplistic nature of this relationship, we forbear to plot the function, although we note that again there are a family of functions, corresponding to the various values of k .

We note that, in contrast to the previous two cases, the transmittance noise depends on the magnitude of the effect and on the transmittance of the sample, but does not depend on the energy of the reference beam; in other words, whereas in the previous two cases the signal-to-noise level of the reference beam was a key factor in determining the behavior of the transmittance noise, here it does not. This conforms to intuition, because when we state that the noise superimposed on the signal is proportional to the signal, the implicit consequence is that the signal-to-noise (or noise-to-signal) is constant.

Let us now, as we normally do, continue to derive the expressions for absorbance noise. Again referring to our previous column (8), we can start with equation 29:

$$Var(\Delta A) = \left(\frac{-0.4343}{E_s}\right)^2 Var(\Delta E_s) + \left(\frac{0.4343}{E_r}\right)^2 Var(\Delta E_r) \quad [29]$$

Again substituting $k^2E_r^2$ and $k^2E_s^2$ for the two variances in Equation 29:

$$Var(\Delta A) = \left(\frac{-0.4343}{E_s}\right)^2 k^2E_s^2 + \left(\frac{0.4343}{E_r}\right)^2 k^2E_r^2 \quad [144]$$

$$\text{Var}(\Delta A) = 2 \times 0.4343^2 k^2 \quad [145]$$

$$\text{SD}(\Delta A) = \sqrt{2} \times 0.4343 k \quad [146]$$

Here again, in the low-noise case of scintillation noise, the absorbance noise is again independent of the reference signal level, and is now independent of the sample characteristics as well, and depends only on the magnitude of the external noise source.

In conformance with our regular pattern, we now derive the behavior of the relative absorbance noise for the low-noise case. Here we start with equation 100, the derivation of which is found in reference 9:

$$\text{Var}\left(\frac{\Delta A}{A}\right) = \frac{1}{(E_s \ln(T))^2} \text{Var}(\Delta E_s) + \frac{1}{(E_r \ln(T))^2} \text{Var}(\Delta E_r) \quad [100]$$

And once more substituting $k^2 E_r^2$ and $k^2 E_s^2$ for the two variance terms:

$$\text{Var}\left(\frac{\Delta A}{A}\right) = \frac{1}{(E_s \ln(T))^2} k^2 E_s^2 + \frac{1}{(E_r \ln(T))^2} k^2 E_r^2 \quad [147]$$

$$\text{Var}\left(\frac{\Delta A}{A}\right) = \frac{2k^2}{(\ln(T))^2} \quad [148]$$

$$\text{SD}\left(\frac{\Delta A}{A}\right) = \frac{\sqrt{2}k}{\ln(T)} \quad [149]$$

Equation 149 presents a minor difficulty; one that is easily resolved, however, so let us do so: the difficulty actually arises in the step between equations 148 and 149, the taking of the square root of the variance to obtain the standard deviation. Conventionally, we ordinarily take the positive square root. However, T takes values from zero to unity; that is, it is always less than unity. The logarithm of a number less than unity is negative; hence, under these circumstances, the denominator of equation 149 would be negative, which would lead to a negative value of the standard deviation. But a standard deviation must always be positive; clearly then, in this case we must use the negative square root of the variance to compute the standard deviation of the relative absorbance noise.

In Figure 30 we plot the function $-1/\ln(T)$ to complete this part of the analysis. We note that there is no minimum to the curve, and the noise from source continually improves as the transmittance decreases. In this case, the previous, conventional derivations agree with our results, although

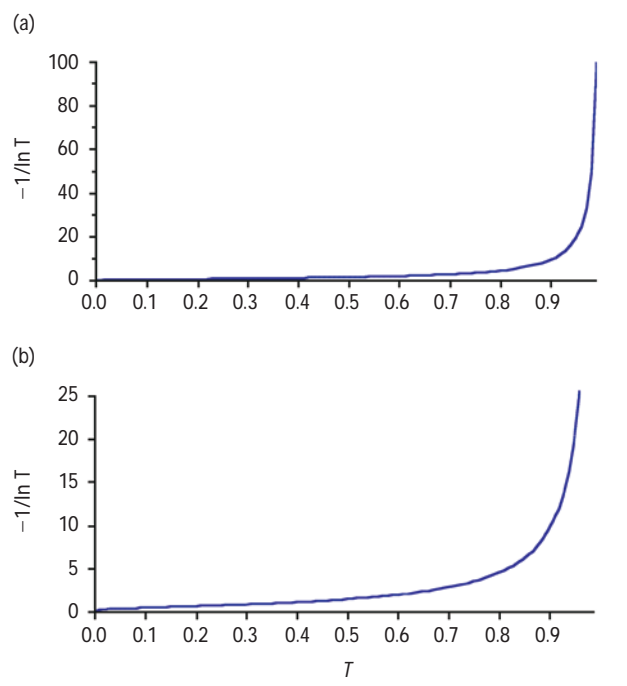


Figure 30. Plot of $-1/\ln(T)$; 30(b) is an ordinate expansion of 30(a).

they do not indicate the $2^{1/2}$ factor. Noting the transitions from equations 140 to 142 (and the corresponding portions of the derivation for absorbance noise and relative absorbance noise), we see that this factor arises from the equal noise contributions of the sample and reference channels; therefore we conclude that in this case also, the missing factor is due to the neglect of the reference channel noise contribution.

The rate of increase in noise also increases faster as T increases (not surprising for a logarithmic function), so that working at transmittance values less than, say, 0.7 or 0.8 is prudent. Of course, we must also remember that our derivations are idealizations, and as Ingle and Crouch point out (see reference 13, page 153), in a real measurement situation, at some point another noise source would become dominant and limit the actual noise observed.

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