

Practical Interference Alignment and Cancellation for MIMO Underlay Cognitive Radio Networks with Multiple Secondary Users

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Abstract—In the underlay cognitive radio approach, secondary users (SUs) are allowed to transmit their messages as long as the negative impact on the performance of the primary user (PU) is below a certain threshold. We consider a MIMO underlay cognitive radio network with a single PU and multiple SUs. We propose a practical interference alignment and cancellation scheme that not only avoids interference at the PU, but also optimizes SUs' performance in terms of degrees of freedom. Given any SU, the interference from the other SUs is aligned, and the interference at the SU from the PU is cancelled in the spatial domain. We give the feasibility condition for achieving interference alignment and cancellation. Due to the NP-hardness of the problem, we resort to approximation algorithms. We also consider practical issues for implementation. The effectiveness of the proposed scheme is shown by simulation results.

Keywords—Cognitive radio, underlay, MIMO, interference alignment, blind null space learning.

I. INTRODUCTION

There is a growing spectrum shortage problem due to limited availability of unallocated spectrum and the ever increasing demand for spectrum from emerging wireless applications. Cognitive radio is a promising technology to solve the spectrum shortage problem by allowing secondary users (SUs) to coexist with primary users (PUs). Three main approaches to dynamic spectrum access have been investigated: interweave, underlay and overlay [1]. In this paper, we consider the underlay approach, which allows an SU to transmit when the interference to the PU receiver is kept below an acceptable threshold. For convenience, we define a user as a transmitter-receiver pair, where the transmitter communicates with the receiver. For a PU, we call the transmitter and the receiver *PU transmitter* and *PU receiver*, respectively. We can define the SU transmitter and SU receiver in the same way.

Much attention has been paid to Multiple Input Multiple Output (MIMO) communications in cognitive radio networks recently, especially for underlay cognitive radios. With the spatial dimensions provided by multiple antenna techniques, the SU may align its signals to the null space of the interference channel from the SU transmitter to the PU receiver. However, without the cooperation of the PU, especially in the case where the PU does not provide explicit feedback to the SU

transmitter, sophisticated mechanisms are needed to estimate the null space of the interference channel.

In [2], a null space sensing scheme is proposed for the MIMO underlay cognitive radio network by assuming channel reciprocity between the SU transmitter and the PU receiver. The SU learns from the signal of the PU, and estimates the null space of the interference channel from the second order statistics of the signal. Because of the assumption of channel reciprocity, this scheme is applicable to time division duplexing (TDD) only. Later, a Blind Null-Space Learning (BNSL) algorithm that does not require the cooperation by the PU is proposed in [3], where the reciprocity of the channel between the SU transmitter and the PU receiver is not required. Under the assumptions that the PU transmitter applies power control to adapt the transmit power to keep the SINR at the PU receiver unchanged, and that the SU transmitter is able to sense the change in the transmit power of the PU, the SU may estimate the null space of the interference channel blindly. However, the BNSL algorithm only tries to avoid the SU interference at the PU.

In this paper, we design an interference alignment and cancellation scheme that not only avoids SU interference at the PU, but also optimizes SUs' performance in terms of degrees of freedom. To optimize the SU performance, for any SU, we minimize the interference from the other SUs by employing interference alignment subject to the constraint that the SU transmissions do not affect the PU. In addition, we minimize the interference from the PU to the SU by cancelling the interference in the spatial domain. We present the feasibility conditions for the interference alignment and cancellation problem. We show that the effect of the requirements of eliminating the interference from the PU to the SUs and the interference from the SUs to the PU is to reduce the number of antennas at each SU transmitter by the number of antennas at the PU receiver, and the number of antennas at each SU receiver by the number of antennas at the PU transmitter. Due to the NP-hardness of the interference alignment and cancellation problem, we resort to approximation algorithms by leveraging prior algorithms[4]. Our proposed scheme can be considered as an underlay approach because of the existence of interference (although insignificant) to the PU during the blind null space learning process.

The remainder of the paper is organized as follows: In Section II, we describe the system model. In Section III, we propose the practical interference alignment and cancellation scheme. In Section IV, we present simulation results. We conclude the paper in Section V.

II. SYSTEM MODEL

Consider a cognitive radio network with one PU transmitter-receiver pair, and K SU transmitter-receiver pairs. The SUs are allowed to transmit, only when the resulting interference at the PU receiver is below a given threshold. The PU does not cooperate with the SUs, while the SUs may cooperate with each other to achieve better performance. Let the PU transmitter and receiver be equipped with M_p and N_p antennas, respectively, and SU transmitter k and receiver k be equipped with M_k and N_k antennas, respectively, $\forall k = 1, \dots, K$. The channel model for the PU receiver can be expressed as:

$$\mathbf{Y}_p = \mathbf{H}_{pp}\mathbf{X}_p + \sum_{k=1}^K \mathbf{H}_{pk}\mathbf{X}_k + \mathbf{Z}_p, \quad (1)$$

where $\mathbf{Y}_p \in \mathbf{C}^{N_p \times 1}$ is the received signal vector at the PU receiver; $\mathbf{H}_{pp} \in \mathbf{C}^{N_p \times M_p}$ and $\mathbf{H}_{pk} \in \mathbf{C}^{N_p \times M_k}$ are the channel matrix between the PU transmitter and the PU receiver, and the channel matrix between the k -th SU transmitter and the PU receiver, respectively, whose entries are assumed i.i.d. with circular normal distribution $\mathcal{CN}(0, 1)$; $\mathbf{X}_p \in \mathbf{C}^{M_p \times 1}$ and $\mathbf{X}_k \in \mathbf{C}^{M_k \times 1}$ are the signal vector transmitted by the PU transmitter and the k -th SU transmitter, respectively; $\mathbf{Z}_p \in \mathbf{C}^{N_p \times 1}$ is an additive white Gaussian noise vector with i.i.d. $\mathcal{CN}(0, 1)$ entries. Similarly, the received signal at the j -th SU receiver is as follows:

$$\mathbf{Y}_j = \mathbf{H}_{jp}\mathbf{X}_p + \sum_{k=1}^K \mathbf{H}_{jk}\mathbf{X}_k + \mathbf{Z}_j, \quad j = 1, \dots, K. \quad (2)$$

III. THE PROPOSED SCHEME

We first formulate the interference alignment and cancellation problem, and then describe how to obtain the null spaces of the channels from the SUs to the PU in practice, and finally investigate the feasibility conditions and describe how to solve the problem in practice and implementation considerations.

A. Problem Formulation

For the MIMO cognitive radio network in (1) and (2), we set $\mathbf{X}_k = \mathbf{V}_k \mathbf{s}_k$, where \mathbf{V}_k is the $M_k \times d_k$ transmit matrix at the k -th SU transmitter, and \mathbf{s}_k the $d_k \times 1$ symbol vector transmitted by the k -th SU with i.i.d elements and an identity covariance matrix. That is, \mathbf{s}_k does not necessarily follow a normal distribution, which makes a slight generalization of the assumption in [4]. Furthermore, we assume that the columns of \mathbf{V}_k are orthonormal, i.e., $\mathbf{V}_k^* \mathbf{V}_k = \mathbf{I}_{d_k}$. Let \mathbf{U}_k be the $N_k \times d_k$ interference cancellation matrix at the k -th SU receiver, where the columns of \mathbf{U}_k are orthonormal, i.e., $\mathbf{U}_k^* \mathbf{U}_k = \mathbf{I}_{d_k}$. Accordingly, we may rewrite equations (1) and (2) as follows,

$$\mathbf{Y}_p = \mathbf{H}_{pp}\mathbf{X}_p + \sum_{k=1}^K \mathbf{H}_{pk}\mathbf{V}_k\mathbf{s}_k + \mathbf{Z}_p \quad (3)$$

$$\mathbf{U}_j^* \mathbf{Y}_j = \mathbf{U}_j^* \mathbf{H}_{jp} \mathbf{X}_p + \sum_{k=1}^K \mathbf{U}_j^* \mathbf{H}_{jk} \mathbf{V}_k \mathbf{s}_k + \mathbf{U}_j^* \mathbf{Z}_j \quad (4)$$

The interference alignment and cancellation problem (called Problem-1) is then: given degrees of freedom (d_1, \dots, d_K) , determine the matrices \mathbf{V}_k and \mathbf{U}_k such that the transmission of the PU is not affected by the SUs, i.e. $\mathbf{H}_{pk} \mathbf{V}_k = \mathbf{0}, \forall k = 1, 2, \dots, K$, where $\mathbf{0}$ is an all-zero matrix of appropriate size; and the interference at an SU caused by the PU and other SUs is totally eliminated by the interference cancellation matrix, i.e., $\mathbf{U}_j^* \mathbf{H}_{jp} = \mathbf{0}$, and $\mathbf{U}_j^* \mathbf{H}_{jk} \mathbf{V}_k = \mathbf{0}, \forall j \neq k = 1, 2, \dots, K$, where $*$ stands for transpose and complex conjugate.

If we choose a feasible K -tuple (d_1, \dots, d_K) that maximizes the sum of degrees of freedom $\sum_{k=1}^K d_k$, then the solution to the above problem becomes an optimization problem. In fact, the interference alignment feasibility conditions to be discussed in Section III-C can be used to determine such d_k 's.

The requirement $\mathbf{H}_{pk} \mathbf{V}_k = \mathbf{0}$ implies that the SU transmissions $\mathbf{X}_k = \mathbf{V}_k \mathbf{s}_k$ are in the null space of \mathbf{H}_{pk} , denoted as $\mathcal{N}(\mathbf{H}_{pk}) = \{\mathbf{v} \in \mathbf{C}^{M_k \times 1} : \mathbf{H}_{pk} \mathbf{v} = \mathbf{0}\}$, which is nontrivial (i.e., containing more than the all zero vector) if $M_k > N_p$. A challenge in the design is how an SU can determine the null space given that the PU does not cooperate with the SUs, and may not even be aware of the existence of the SUs. Recently, a blind null-space learning (BNSL) algorithm is proposed to determine the null spaces[3], which we briefly describe below.

B. Blind Null-Space Learning

The BNSL algorithm is based on two assumptions: (1) that the PU transmitter adapts its transmission power to maintain the required SINR at the PU receiver in the event of SU interference, and (2) that the SU transmitters can detect changes in the transmission power of the PU. The main idea is based on the fact that the null space of \mathbf{H}_{pk} is the same as the null space of $\mathbf{G}_{pk} = \mathbf{H}_{pk}^* \mathbf{H}_{pk}$, which is then determined by blind Jacobi eigenvalue decomposition without observing \mathbf{G}_{pk} itself nor the rotated \mathbf{G}_{pk} in the iterative Jacobi eigenvalue decomposition process, hence the name "blind". The BNSL algorithm begins with $\mathbf{A}_0 = \mathbf{G}_{pk}$, and each iteration of the algorithm contains $M_k(M_k - 1)/2$ learning stages. In each learning stage, the algorithm performs line searches [3] by sending different training signals to obtain a rotation matrix \mathbf{R}_l in order to update the matrix \mathbf{A}_l as $\mathbf{A}_{l+1} = \mathbf{R}_l^* \mathbf{A}_l \mathbf{R}_l$, such that two off-diagonal elements of \mathbf{A}_{l+1} are eliminated. After an iteration of $M_k(M_k - 1)/2$ learning stages, every off-diagonal element of \mathbf{G}_{pk} is eliminated once. The algorithm may perform several iterations to improve the accuracy, and at the end, we obtain the matrix $\mathbf{A} = \mathbf{R}^* \mathbf{G}_{pk} \mathbf{R}$, where the off-diagonal elements of \mathbf{A} are close to 0 and \mathbf{R} is the multiplication of all rotation matrices \mathbf{R}_l . Consequently, \mathbf{R} is an estimate of the eigenspace of \mathbf{G}_{pk} , while the null space of \mathbf{G}_{pk} is the eigenvector space corresponding to the eigenvalues equal to 0. The BNSL algorithm above is for single SU case. However, it can be easily generalized to multiple secondary user systems, if in the line searches only one SU changes its training signal at each time.

Once the SUs learn the null spaces of channels \mathbf{H}_{pk} , we can satisfy the design constraint that the transmissions from the

SU do not affect the PU by restricting the SU transmissions to be within the null spaces.

C. Feasibility Conditions

We first consider the received signal in (3) at the PU receiver. By applying the BNSL algorithm described earlier, each SU can obtain its null space $\mathcal{N}(\mathbf{H}_{pk})$. If we select the columns of the beamforming matrix \mathbf{V}_k from the null space for each SU, the interference at the PU receiver will be completely removed since $\mathbf{H}_{pk}\mathbf{V}_k = \mathbf{0}, \forall k$. In other words, the existence of the SUs will not affect the PU. Note that to find the required \mathbf{V}_k , the dimensions of the null space should not be less than the dimensions of the signal vector, i.e., $M_k - N_p \geq d_k$. In order to guarantee $\mathbf{H}_{pk}\mathbf{V}_k = \mathbf{0}, \forall k$, we let $\mathbf{V}_k = \mathbf{B}_k\mathbf{P}_k$, where \mathbf{B}_k is a $M_k \times (M_k - N_p)$ matrix and the column vectors of \mathbf{B}_k form an orthonormal basis of the null space $\mathcal{N}(\mathbf{H}_{pk})$, and where \mathbf{P}_k is a $(M_k - N_p) \times d_k$ matrix such that $\mathbf{P}_k^*\mathbf{P}_k = \mathbf{I}_{d_k}$.

We then consider the received signal in (4) at the SU receivers. The signal sent by the PU transmitter now serves as the interference to the SUs. In practice, if the SU receiver knows the training sequence of the PU, it can estimate the interference channel \mathbf{H}_{jp} by overhearing the transmission of the training sequence from the PU transmitter to the PU receiver. Assuming that the interference channel \mathbf{H}_{jp} is known at the j th SU receiver, the j th SU receiver may apply an $N_j \times (N_j - M_p)$ interference suppression matrix \mathbf{W}_j , satisfying $\mathbf{W}_j^*\mathbf{H}_{jp} = \mathbf{0}$, to eliminate the interference from the PU as follows:

$$\mathbf{W}_j^*\mathbf{Y}_j = \sum_{k=1}^K \mathbf{W}_j^*\mathbf{H}_{jk}\mathbf{V}_k\mathbf{s}_k + \mathbf{W}_j^*\mathbf{Z}_j, \quad (5)$$

where the column vectors of \mathbf{W}_j are orthonormal. Such \mathbf{W}_j can be constructed from the null space of \mathbf{H}_{jp}^* , because the requirement $\mathbf{W}_j^*\mathbf{H}_{jp} = \mathbf{0}$ is equivalent to $\mathbf{H}_{jp}^*\mathbf{W}_j = \mathbf{0}$. Since the entries of \mathbf{H}_{jp} are drawn from a continuous distribution, its columns are linearly independent with probability 1 if $N_j \geq M_p$. Note that to find such \mathbf{W}_j , the null space of \mathbf{H}_{jp} must be nontrivial, i.e., the dimension of the null space $N_j - M_p > 0$, which automatically satisfies the prior requirement $N_j \geq M_p$.

For convenience, let $\tilde{\mathbf{Y}}_j = \mathbf{W}_j^*\mathbf{Y}_j$, $\tilde{\mathbf{Z}}_j = \mathbf{W}_j^*\mathbf{Z}_j$, and

$$\tilde{\mathbf{H}}_{jk} = \mathbf{W}_j^*\mathbf{H}_{jk}\mathbf{B}_k, \quad (6)$$

where we recall that $\mathbf{W}_j \in \mathcal{N}(\mathbf{H}_{jp}^*)$ and $\mathbf{B}_k \in \mathcal{N}(\mathbf{H}_{pk})$. Then the received signal at the j -th SU can be rewritten as

$$\tilde{\mathbf{Y}}_j = \sum_{k=1}^K \tilde{\mathbf{H}}_{jk}\mathbf{P}_k\mathbf{s}_k + \tilde{\mathbf{Z}}_j, \quad j = 1, \dots, K, \quad (7)$$

which can be regarded as an interference channel of a network of K SUs and zero PUs. In this interference channel, the channels $\tilde{\mathbf{H}}_{jk} = \mathbf{W}_j^*\mathbf{H}_{jk}\mathbf{B}_k$ automatically guarantee that both the interference from the SUs to the PU and the interference from the PU to the SUs are eliminated. The j th SU receiver can further apply a $(N_j - M_p) \times d_j$ interference suppression matrix \mathbf{D}_j that satisfies $\mathbf{D}_j^*\mathbf{D}_j = \mathbf{I}_{d_j}$, resulting in

$$\mathbf{D}_j^*\tilde{\mathbf{Y}}_j = \sum_{k=1}^K \mathbf{D}_j^*\tilde{\mathbf{H}}_{jk}\mathbf{P}_k\mathbf{s}_k + \mathbf{D}_j^*\tilde{\mathbf{Z}}_j, \quad (8)$$

where the effective noise $\mathbf{D}_j^*\tilde{\mathbf{Z}}_j = \mathbf{D}_j^*\mathbf{W}_j^*\mathbf{Z}_j$ follows the same distribution as \mathbf{Z}_j because the columns of $\mathbf{W}_j\mathbf{D}_j$ are orthonormal.

The interference alignment and cancellation problem (described right after (4)) is now reduced to the following standard interference alignment problem (called Problem-2): given degrees of freedom (d_1, \dots, d_K) , find precoding matrices \mathbf{P}_k and decoding matrices \mathbf{D}_k

$$\mathbf{P}_k : (M_k - N_p) \times d_k, \quad \mathbf{P}_k^*\mathbf{P}_k = \mathbf{I}_{d_k}, \forall k = 1, \dots, K, \quad (9)$$

$$\mathbf{D}_k : (N_k - M_p) \times d_k, \quad \mathbf{D}_k^*\mathbf{D}_k = \mathbf{I}_{d_k}, \forall k = 1, \dots, K, \quad (10)$$

such that

$$\mathbf{D}_j^*\tilde{\mathbf{H}}_{jk}\mathbf{P}_k = \mathbf{0}_{d_j \times d_k}, \forall k \neq j \quad (11)$$

$$\text{rank}(\mathbf{D}_j^*\tilde{\mathbf{H}}_{jj}\mathbf{P}_j) = d_j, \forall j = 1, \dots, K, \quad (12)$$

where $\tilde{\mathbf{H}}_{jk}$ are defined in (6).

Note that the effect of the requirement of eliminating the interference from the PU to the SUs and the interference from the SUs to the PU is to reduce the numbers of antennas of the SUs. Specifically, for any SU j , $j = 1, \dots, K$, the number of antennas M_j at the transmitter is reduced by the number of PU receive antennas N_p , and the number of antennas N_j at the receiver is reduced by the number of PU transmit antennas M_p .

If we are able to obtain \mathbf{P}_k and \mathbf{D}_k that solve the interference alignment problem, then we can immediately determine $\mathbf{V}_k = \mathbf{B}_k\mathbf{P}_k$ and $\mathbf{U}_k = \mathbf{W}_k\mathbf{D}_k$ for (3) and (4), respectively, for the interference alignment and cancellation problem.

Now we consider the feasibility question, i.e., whether there exist matrices \mathbf{P}_k and \mathbf{D}_k that satisfy the interference alignment problem (9) – (12). The feasibility conditions for the interference alignment problem (9) – (12) are only obtained for some special systems [5], [6], [7] and these results can be carried over to here. As an example, in [7], it is shown that in an interference network (without PU of course), if we assume that $M_k = M$, $N_k = N$ and $d_k = d$ for all k , and M and N are divisible by d , then interference alignment is feasible if and only if $M + N \geq d(K + 1)$. For the problem at hand, consider a special case where $M_k = M_s$, $N_k = N_s$, $d_k = d_s, \forall k = 1, \dots, K$, with the subscript s indicates secondary users. Invoking the result in [7], we have that if $(M_s - N_p)$ and $(N_s - M_p)$ are divisible by d , then the interference alignment problem (9) – (12) and hence the interference alignment and cancellation problem are feasible if and only if $(M_s + N_s - M_p - N_p) \geq d(K + 1)$.

D. Approximation Algorithms

Another question about the interference alignment problem, Problem-2 (see (9) – (12)), is the practicality of finding the solution numerically by a computer, i.e., whether there exists a computationally efficient algorithm that determines matrices \mathbf{P}_k and \mathbf{D}_k solving the interference alignment problem (9) – (12). It has been shown [8] that this problem is NP-hard in the number of SUs K . Therefore, the bigger problem, Problem-1, is also NP-hard. In practice, we can only resort to approximation algorithms. The details of the practical interference alignment and cancellation algorithm are shown in Algorithm 1.

The first approximation algorithm that we use to solve Problem-2 (see (9) – (12)) is the iterative interference alignment (IIA) algorithm in [4]. The key idea in [4] is to construct a reciprocal interference channel such that the interference alignment condition for the reciprocal interference channel is the same as that of the original interference channel. In the reciprocal interference channel, the roles of the transmitters and receivers are switched, and $\bar{\mathbf{H}}_{kj} = \tilde{\mathbf{H}}_{jk}^*$ is the channel matrix from the j -th SU receiver to the k -th SU transmitter. Note that the reciprocal interference channel is merely a theoretical apparatus, and it has nothing to do with whether the physical channels are reciprocal or not. Denote by $\bar{\mathbf{P}}_k$ and $\bar{\mathbf{D}}_k$ the transmit and receive matrices of the reciprocal interference channel, respectively. Let $\bar{\mathbf{P}}_k = \mathbf{D}_k$ and $\bar{\mathbf{D}}_k = \mathbf{P}_k$, and we have $\bar{\mathbf{D}}_k^* \bar{\mathbf{H}}_{kj} \bar{\mathbf{P}}_j = (\mathbf{D}_j^* \tilde{\mathbf{H}}_{jk} \mathbf{P}_k)^*$. Thus, the interference alignment of the reciprocal interference channel is feasible as long as that of the original one is feasible and vice versa, and the transmit and receive matrices can be obtained by exchanging those obtained on the original one.

The IIA algorithm begins with arbitrary precoding matrices \mathbf{P}_k such that $\mathbf{P}_k^* \mathbf{P}_k = \mathbf{I}$. In each iteration, the k th SU receiver computes the interference covariance matrix

$$\mathbf{Q}_k = \sum_{j=1, j \neq k}^K \frac{P_j}{d_j} \tilde{\mathbf{H}}_{kj} \mathbf{P}_j \mathbf{P}_j^* \tilde{\mathbf{H}}_{kj}^*, \quad (13)$$

where P_j is the total transmit power of SU transmitter j . With the definition of \mathbf{s}_k in this paper, which slightly generalizes the one in [4] by eliminating the normal distribution assumption, \mathbf{Q}_k is still closely related to the total interference leakage at receiver k [4]. Denote the interference signal by $\mathbf{r}_k = \sum_{j=1, j \neq k}^K \tilde{\mathbf{H}}_{kj} \mathbf{P}_j$. The expected value of the interference power $E[|\mathbf{D}_k^* \mathbf{r}_k|^2] = E[\text{tr}(\mathbf{D}_k^* \mathbf{r}_k \mathbf{r}_k^* \mathbf{D}_k)] = \text{tr}(\mathbf{D}_k^* \mathbf{Q}_k \mathbf{D}_k)$, where we have used the assumption that the covariance matrix of \mathbf{s}_k is an identity matrix. Due to its non-negativity, the interference power approaches zero if its expected value approaches zero. Therefore, the k -th user tries to minimize $\text{tr}(\mathbf{D}_k^* \mathbf{Q}_k \mathbf{D}_k)$, which is done by choosing the columns of \mathbf{D}_k as the eigenvectors of \mathbf{Q}_k corresponding to the smallest d_k eigenvalues such that $\mathbf{D}_k^* \mathbf{D}_k = \mathbf{I}$.

Then reverse the communication direction and consider the reciprocal channel. Set $\bar{\mathbf{P}}_k = \mathbf{D}_k$. Compute the interference covariance matrix

$$\bar{\mathbf{Q}}_k = \sum_{j=1, j \neq k}^K \frac{\bar{P}_j}{d_j} \bar{\mathbf{H}}_{kj} \bar{\mathbf{P}}_j \bar{\mathbf{P}}_j^* \bar{\mathbf{H}}_{kj}^*, \quad (14)$$

where \bar{P}_j is the total transmit power of receiver j (which serves as a transmitter in the reciprocal channel), and choose the columns of $\bar{\mathbf{D}}_k$ as the eigenvectors of $\bar{\mathbf{Q}}_k$ corresponding to the smallest d_k eigenvalues such that $\bar{\mathbf{D}}_k^* \bar{\mathbf{D}}_k = \mathbf{I}$.

Then reverse the communication direction again and consider the original channel. Set $\mathbf{P}_k = \bar{\mathbf{D}}_k$. Begin the next iteration until the algorithm converges.

The IIA algorithm allows for a distributed implementation, because although the interference covariance matrix (13) depends on all channel matrices and precoding matrices, it can be estimated as a whole by the k -th user in a distributed manner [4]. The details of the IIA algorithm are shown in Lines 6-14 in Algorithm 1.

Algorithm 1 Practical Interference Alignment and Cancellation Algorithm for an Underlay Cognitive Radio Network

- Require:** Channel matrices \mathbf{H}_{kp} and \mathbf{H}_{kj} for $k, j = 1, 2, \dots, K$
- Ensure:** Determine transmit beamforming matrices \mathbf{V}_k and receive beamforming matrices \mathbf{U}_k for $k = 1, 2, \dots, K$
- 1: Perform the BNSL algorithm to obtain $\mathcal{N}(\mathbf{H}_{pk})$;
 - 2: Let the columns of $\bar{\mathbf{B}}_k$ be a basis of $\mathcal{N}(\mathbf{H}_{pk})$;
 - 3: Let the columns of $\bar{\mathbf{W}}_k$ be a basis of $\mathcal{N}(\mathbf{H}_{kp}^*)$;
 - 4: $\tilde{\mathbf{Y}}_j = \bar{\mathbf{W}}_j^* \mathbf{Y}_j$, $\tilde{\mathbf{H}}_{jk} = \bar{\mathbf{W}}_j^* \mathbf{H}_{jk} \bar{\mathbf{B}}_k$;
 - 5: **IF** the approximation algorithm is the IIA algorithm
 - 6: Initialize \mathbf{P}_k as a random $(M_k - N_p) \times d_k$ matrix such that $\mathbf{P}_k^* \mathbf{P}_k = \mathbf{I}_{d_k}$;
 - 7: Begin iteration:
 - 8: Compute \mathbf{Q}_k according to (13);
 - 9: Let the columns of \mathbf{D}_k be the eigenvectors corresponding to the smallest d eigenvalues of \mathbf{Q}_k ;
 - 10: Reverse the communication direction, set $\bar{\mathbf{P}}_k = \mathbf{D}_k, \forall k$;
 - 11: Compute $\bar{\mathbf{Q}}_k$ according to (14);
 - 12: Let the columns of $\bar{\mathbf{P}}_k$ be the eigenvectors corresponding to the smallest d_k eigenvalues of $\bar{\mathbf{Q}}_k$;
 - 13: Reverse the communication direction, set $\mathbf{P}_k = \bar{\mathbf{D}}_k, \forall k$;
 - 14: Back to 7 until convergence;
 - 15: **ELSE IF** the approximation algorithm is the Max-SINR algorithm
 - 16: Initialize \mathbf{P}_k as a $M_k \times d_k$ matrix such that the columns are linearly independent;
 - 17: Begin iteration:
 - 18: Compute $\mathbf{T}_{kl}, \forall k, l$ according to (16);
 - 19: Compute $(\mathbf{D}_k)_{*l}, \forall k, l$ according to (15);
 - 20: Reverse the communication direction, set $\bar{\mathbf{P}}_k = \mathbf{D}_k, \forall k$;
 - 21: In the reciprocal network, compute $\bar{\mathbf{T}}_{kl}, \forall k, l$;
 - 22: Compute $(\bar{\mathbf{D}}_k)_{*l}, \forall k$;
 - 23: Reverse the communication direction, set $\mathbf{P}_k = \bar{\mathbf{D}}_k, \forall k$;
 - 24: Back to 17 until convergence;
 - 25: **END IF**
 - 26: $\mathbf{V}_k = \bar{\mathbf{B}}_k \mathbf{P}_k, \mathbf{U}_k = \bar{\mathbf{W}}_k \bar{\mathbf{D}}_k$;
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The second approximation algorithm that we consider to solve Problem-2 is the Max-SINR algorithm[4]. The IIA algorithm tries to align the interferences in a subspace orthogonal to the desired signal subspace, but makes no effort to maximize the desired signal power in the desired signal subspace. This deficiency is addressed by the Max-SINR algorithm. The other aspects of the Max-SINR algorithm are similar to those of the IIA algorithm. Specifically, in the original channel, the l th column of \mathbf{D}_k , denoted by $(\mathbf{D}_k)_{*l}$ is set as follows to maximize the SINR of the l th stream at receiver k

$$(\mathbf{D}_k)_{*l} = \frac{(\mathbf{T}_{kl})^{-1} \mathbf{H}_{kk} (\mathbf{P}_k)_{*l}}{\|(\mathbf{T}_{kl})^{-1} \mathbf{H}_{kk} (\mathbf{P}_k)_{*l}\|}, \quad (15)$$

where $(\cdot)_{*l}$ represents the l th column of matrix (\cdot) , and

$$\begin{aligned} \mathbf{T}_{kl} = & \sum_{j=1}^K \sum_{d=1}^{d_j} \frac{P_j}{d_j} \mathbf{H}_{kj} (\mathbf{P}_j)_{*d} (\mathbf{P}_j)_{*d}^* \mathbf{H}_{kj}^* \\ & - \frac{P_k}{d_k} \mathbf{H}_{kk} (\mathbf{P}_k)_{*l} (\mathbf{P}_k)_{*l}^* \mathbf{H}_{kk}^* + \mathbf{I}_{N_k}, \end{aligned} \quad (16)$$

is the covariance matrix of the interference and noise. In the reciprocal network, set $\bar{\mathbf{P}}_k = \mathbf{D}_k, \forall k$. The interference covari-

ance matrix $\bar{\mathbf{T}}_{kl}$ is calculated by the substitutions $\mathbf{P}_j \leftarrow \bar{\mathbf{P}}_j$, $P_j \leftarrow \bar{P}_j$, $\mathbf{H}_{kj} \leftarrow \bar{\mathbf{H}}_{kj}$ in (16). $(\bar{\mathbf{D}}_k)_{*l}$ is calculated by similar substitutions in (15). The details of the Max-SINR algorithm are shown in Lines 16-24 in Algorithm 1.

Note that both the IIA algorithm and the Max-SINR algorithm converge [4].

E. Practical Considerations for Implementation

We have considered various practical issues for the proposed Algorithm 1. Here, we look at those and others in a more holistic view. The first issue is about the the formulation of the interference alignment and cancellation problem in Section III-A. The design constraint that SUs do not affect the PU is stated as $\mathbf{H}_{pk}\mathbf{V}_k = \mathbf{0}$, where we use an equality. However, in practice, due to the truncation of the digits of real/complex numbers and other various errors (e.g., in channel estimation and power measuring), equality will not be achieved. Therefore, we can replace the equality with an inequality, i.e., $\mathbf{H}_{pk}\mathbf{V}_k \leq \epsilon_k \mathbf{O}_{N_p \times d_k}$, where $\mathbf{O}_{N_p \times d_k}$ is a $N_p \times d_k$ matrix with all entries being one and $\epsilon_k > 0$ is a constant determined by the overall error of the system. Similar modifications can be done for $\mathbf{U}_j^* \mathbf{H}_{jp} = \mathbf{0}$ and $\mathbf{U}_j^* \mathbf{H}_{jk} \mathbf{V}_k = \mathbf{0}$, for $\forall j \neq k = 1, 2, \dots, K$.

The second issue is channel estimation. One of the difficulties for interference alignment in general is the need for knowing the global channel state information (CSI). That is, some device (e.g., an eNB in an LTE system) in the network needs to know all channel matrices \mathbf{H}_{jk} . This may not be as undesirable as it seems, because after all, training is generally done to obtain channels \mathbf{H}_{kk} anyway. During the training for SU k , other SU receivers $j \neq k$ can listen due to the broadcast nature of the wireless medium. This way, a single training event will result in K channel estimates. The coordination of training sequence transmission and listening can be achieved by using a low-overhead protocol, for example, one based on TDM scheduling. Thus, getting an estimate of the global CSI incurs only negligible communication overhead. In the event that such coordination is not possible, we can resort to the distributed implementation of both the IIA algorithm and the Max-SINR algorithm [4], where each SU estimates its own channel matrix in the usual way and estimates the interference covariance matrix as a whole. As discussed earlier, the channel matrices \mathbf{H}_{jp} can be estimated at SU receiver j by overhearing the PU training sequence transmission and knowing the PU training sequence, and \mathbf{H}_{pj} can be estimated via the BNSL algorithm.

The third issue is how the approximation algorithms are run in practice. We reiterate that the use of a reciprocal network is only a theoretical apparatus. The physical channels do not have to be reciprocal. In addition, once an SU transmitter or receiver knows the global CSI, it can run the approximation algorithms itself without incurring any communication overhead. We only need to let one SU transmitter or receiver run the approximation algorithms to obtain matrices \mathbf{V}_k and \mathbf{U}_k and distribute them to the respective SUs. In contrast, in a distributed implementation of the approximation algorithms, channel reciprocity of the physical channel is needed, and in addition, real transmissions occur in each iteration. Therefore, the communication overhead could be large if the convergence rates are slow.

The fourth issue is about the channel coherence time. When the channels are static, the BNSL algorithm, other channel estimation algorithms, and the approximation algorithms will work fine. However, in practice, the channels will vary over time, especially for mobile communications. The BNSL algorithm can be augmented to track slow changes in the channels, as shown recently in [9]. For other channel estimation algorithms, as long as the channel coherence time is much larger than K times of the channel training time, the channel estimates can be accurate. For the approximation algorithms, the convergence time should be much less than the channel coherence time. In this regard, the distributed implementations are more restrictive.

IV. SIMULATION

In this section, we present simulation results for the proposed Algorithm 1. We first run the BNSL algorithm to find out the null space of the channel from each SU transmitter to the PU receiver. We then run the approximation algorithm, which is either the IIA algorithm or the max-SINR algorithm.

We consider the systems with three, four and five SU transmitters and receivers, respectively, i.e., $K = 3, 4$ or 5 , and let $M_p = N_p = 2$, $M_k = M_s = 6$ and $N_k = N_s = 4$ for all k . Two and one independent information streams are sent by the PU and each SU, respectively. For convenience, we assume that the transmit powers of the PU and the SUs are identical. In the figures, we show the performance of the PU as solid lines and those of the SUs as dashed lines.

Fig.1 shows the sum rates of the PU and the SUs when the IIA algorithm is applied. The sum rates, when the max-SINR algorithm is applied, are shown in Fig.2. We also show the average rate of a single SU as dotted lines. The numerical results are averaged over 100 channel realizations. Fig.3 and 4 show the bit error rates (BER) of the PU and the SUs using the IIA algorithm and the max-SINR algorithm, respectively. The BPSK constellation is used for the simulations. For reference, we also show the BER performance of the PU, when no SU exists.

While both the rate and BER performances of the PU remain almost the same, the performances of the SUs degrade as the number of SUs increases. As discussed earlier, the interference alignment and cancellation is feasible if and only if $M_s + N_s - M_p - N_p \geq d(K + 1)$. Although the setup satisfies the feasibility condition, we observe from Fig.1 and 3 that interference alignment fails with $K = 5$, probably because of the weakness of the IIA algorithm.

Compared to the IIA algorithm, the max-SINR algorithm always gives better rate and BER performances, even when the interference alignment fails at $K = 5$. However, it requires more computation than the IIA algorithm to converge. For example, in the case that $SNR = 8$ dB and $K = 4$, the average time to converge (in practice we decide that the SINR converges if SINR as a function of the number of iterations flattens out) for the max-SINR algorithm is 2.35 times as long as that for the IIA algorithm with the same termination criterion and the same simulation environment.

V. CONCLUSION

We consider a MIMO underlay cognitive radio network with a single PU and multiple SUs. We propose a practical interference alignment and cancellation scheme that not only avoids interference at the PU, but also optimizes SUs' performance in terms of degrees of freedom. For any SU, the interference from the other SUs is aligned, and the interference from the PU to the SU is cancelled in the spatial domain. We give the feasibility condition for achieving interference alignment and cancellation. Due to the NP-hardness of the problem, we resort to approximation algorithms. We also consider practical issues for implementation. The effectiveness of the proposed scheme is shown by simulation results.

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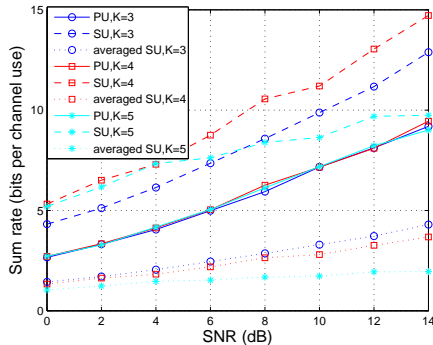


Fig. 1. Sum rates of PU and SUs using the IIA algorithm, with $K=3,4$ and 5.

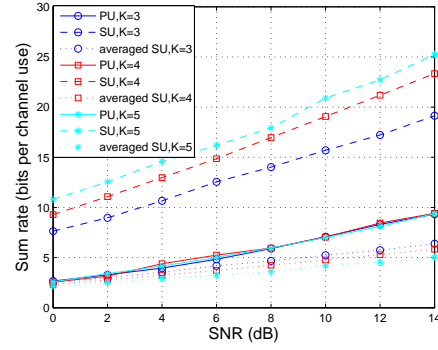


Fig. 2. Sum rates of PU and SUs using the max-SINR algorithm, with $K=3,4$ and 5.

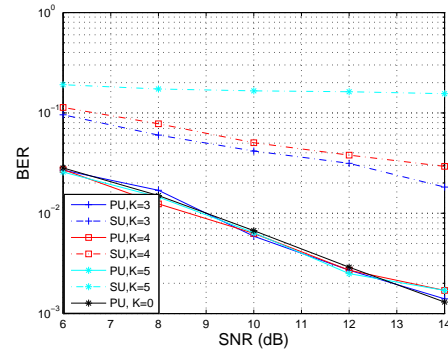


Fig. 3. BERs of PU and SUs using the IIA algorithm, with $K=3,4$ and 5.

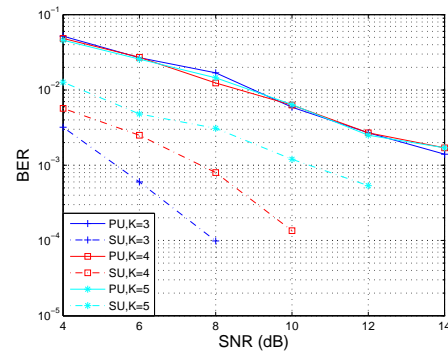


Fig. 4. BERs of PU and SUs using the max-SINR algorithm, with $K=3,4$ and 5.