

Annotated Examples: M155 Right Triangle Trigonometry

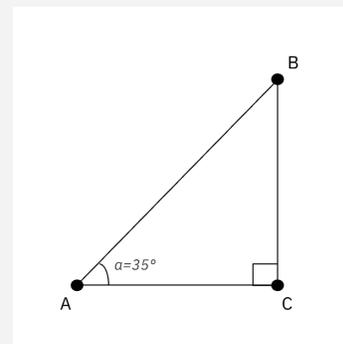
The examples that follow are intended to illustrate how the learning principles are used to support students' engagement with the content and practices outlined in this badge. These examples do not provide comprehensive coverage of those expectations, but rather elevate some of the learning principles that are less likely to be part of published curricular materials for mathematics instruction. The examples that follow were developed by the Math Badging writing team, unless otherwise specified. These are a small sample of types of learning activities that can be done with students, both in and out of a traditional classroom setting.

Example 1: Everything is All Right

Students are given a task like this:

Right triangles have surfaced in previous work: slope, finding areas of complex shapes, and indirect measurement. In this task we will explore ratios of sides of right triangles in order to expand ways we can work with right triangles.

1. Use measurement tools or dynamic geometry software such as GeoGebra or Desmos to construct a right triangle ABC , with a right angle at B and a 35° angle at A .
2. Here is some terminology used to describe the relationship between the sides of a right triangle and the reference angle, which in this case is angle A .
 - Side \overline{AB} is the hypotenuse of this right triangle,
 - Side \overline{BC} is the side opposite angle A , and
 - Side \overline{AC} is the side adjacent to angle A .



Take a moment to study the image provided. What makes sense to you about how each side is referred to?

3. Measure the three sides of the triangle and record the lengths in a table. Please use the terminology offered above to communicate measures.
4. Find each of the ratios listed below.
 - a. $\frac{\text{length of leg opposite angle } A}{\text{length of leg adjacent to angle } A}$
 - b. $\frac{\text{length of leg opposite angle } A}{\text{length of the hypotenuse}}$

c. $\frac{\text{length of leg adjacent angle } A}{\text{length of the hypotenuse}}$

5. If you were to compare the side measurements you found with that of your classmates, how do you think those measurements would compare? Are there any relationships between the side lengths you anticipate would still hold? Why does that make sense?
6. If you were to compare the ratios you found with that of your classmates, how do you think those ratios would compare? Why does that make sense?
7. Do you think the relationships you've noticed hold true for any right triangle? Construct additional right triangles to demonstrate your conjecture.

(Adapted from Interactive Math Program Year 1 Shadows Unit)

Sample Learning Activity

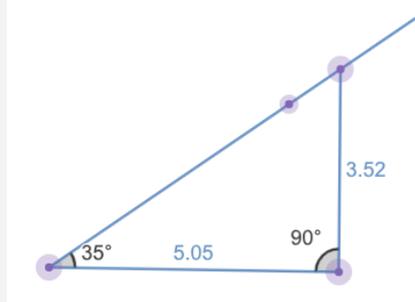
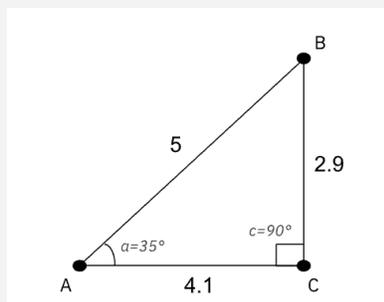
Begin by giving students quiet time to make sense of the information given and process the task they have to complete. Provide them with physical tools, such as graphing paper, transparencies or patty paper, paper, ruler and protractor, and/or access to dynamic geometry software such as [GeoGebra](#) or [Desmos](#).

After some individual time, encourage students to discuss progress with a partner or small group. Encourage students to make their thinking visible.

Initially, focus a conversation on how students went about constructing their triangles. Consider using some of the following questions to facilitate the discussion:

- What strategies led to success in your effort to construct a right triangle with the given conditions?
- What new insights do you have about the work of constructing right triangles?
- If students used dynamic software to complete construction, select students with varied approaches to this work. Look for students who used construction tools to ensure they had 90° and 35° angles, as well as students who may have not controlled for that. Begin the conversation by asking students, “*What happens if we drag some of these points?*”
 - Invite students to attempt to drag points to recognize what elements of their triangle remain constant and which don't.
 - Follow up with additional questions to invite students to reflect on their construction approach; for example, some approaches make it so that either or both of the 90° and 35° angles remain constant.

The image on the left illustrates a student's approach using GeoGebra. The image on the right illustrates a student's approach using Desmos.



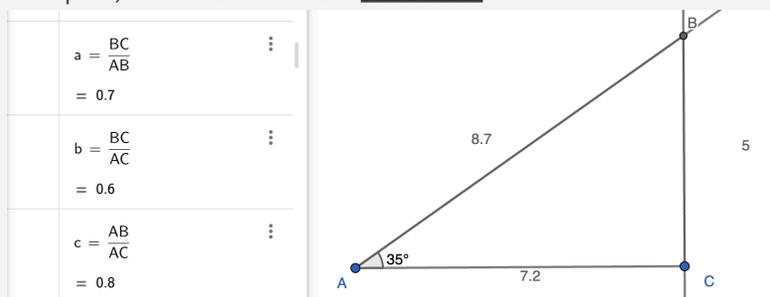
Give students time to revise their constructions as needed before proceeding to the next stage of the investigation. As students enter the next stage of the investigation, hold space for students to have some independent work time before they collaborate with classmates.

For the second prompt, consider using the [Math Language Routine 1: Stronger and Clearer Each Time](#) (MLR 1) to give students an opportunity to learn from each other. The prompt in focus for this iteration of MLR 1 is: *When given a right triangle and an angle to focus on, how can you tell which leg of the triangle is the leg opposite of the angle? ...leg adjacent to the angle? ...hypotenuse?* A possible way to consolidate this conversation is to use [Math Language Routine 2: Collect and Display](#) (MLR 2) or create an anchor poster using student language.

For the last prompts, consider creating a poster where students can contribute their data, similar to the table below, as classmates continue to work through the rest of the prompts.

Student	$\frac{\text{length of leg opposite } A}{\text{length of leg adjacent } A}$	$\frac{\text{length of leg opposite } A}{\text{length of hypotenuse}}$	$\frac{\text{length of leg adjacent } A}{\text{length of hypotenuse}}$

As students develop responses for Questions 5 and 6, monitor for, select, and share student work that embodies different approaches and levels of formality for discussion. Note and select various levels of formality students are comfortable with as they share their work with each other. Look for students who use arguments grounded in the idea of similarity, as well as students who utilize technology to generate multiple examples, as illustrated in the [GeoGebra](#) screenshot below.



As students share their reasoning with each other, consider the following questions:

- *What are some things you appreciate about this explanation?*
- *What questions does this explanation surface for you?*
- *What can you take from this explanation to apply to your own work?*

To engage students in a discussion of Question 7, consider selecting students who constructed additional right triangles with different reference angles, as well as those who approached this by focusing on the measure of $\angle B$ being 55° .

Following the sharing and discussion, give students an opportunity to revise their work and share with each other again. Focus this second discussion on changes or adjustments that they have made based on their peers' work.

In this example, students are:

- given a cognitively demanding task. They are asked to construct right triangles that satisfy given criteria to generate the three basic trigonometric ratios: sine, cosine, and tangent. Students then have an opportunity to justify why these ratios will be approximately the same when compared to classmate's results (LP 1).
- engaging with their peers to describe the rationale behind the terminology used when describing side lengths of a right triangle, relative to the given angle (LP 2).
- building conceptual understanding as they engage in reasoning on why trigonometric ratios of a given triangle remain the same, regardless of the lengths of the sides of the triangle. This builds the foundation for understanding that trigonometric ratios, themselves, are a function of an angle (LP 3).
- encouraged to have agency, as they choose their solution pathways and are given space to reflect on how their approach might change based on engagement with their peers' work (LP 4).
- given an opportunity to see themselves as mathematicians, as their methods for generating data to support conjectures are central to this investigation (LP 5).
- given an opportunity to use geometry software to experiment with changes to the side lengths of the triangle or the given different reference angles (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Interpreting Data:** [Data visualization](#) (FL.MST.3.c)

Example 2: Too Tall to Measure

Students are given a scenario like this:

Our phones offer tools that we can use to measure things without the use of traditional tools, like measuring tape or yard sticks. Today, you will determine the accuracy of such tools. You will need some classmates to work with, a cell phone, and access to a measuring app. This short article, [10 Best Measurement Apps for Android and iPhone](#), by [Beebom](#) offers some options.

Your task:

- 1.** *Select an object to use for this task. Consider a flagpole, bell tower, or basketball basket.*
- 2.** *Use a smartphone's measurement tool to measure the height of the object from at least three different positions.
 - a.** *How do the measurements compare? If there is a difference in the heights, what do you think accounts for the differences you see?**
- 3.** *Determine the height of that object using more traditional techniques or research. How do the measures you got from using the phone compare to the numbers you got from indirect measurement using traditional tools or from your research?*
- 4.** *Select other objects to go through this process.*
- 5.** *Based on your findings, how would you describe the accuracy of the smartphone's measurement app?*
- 6.** *Conduct some additional research to determine how these phone apps work. What recommendations do you have for changes or improvements?*

Prepare to report your findings.

Sample Learning Activity

Based on what you know about your students, you may choose to launch this task by asking students, “*Who has used a measurement app on their phone?*” Following this question, invite a student or two who has used such tools to demonstrate how they work. Alternatively, you can launch this task by showing this short video clip: “[Did you know your Smartphone could do THIS!?](#)” by [@Mrwhosethe boshShorts](#). Share with students that their task is to determine the accuracy of their smartphone measuring tool.

As students work on the first two prompts of this task, monitor and select students or student groups to share their preliminary approach to this task with the whole group. Look for groups who are deliberate about the horizontal positions they are using, as well as students who take into account their ability to frame the object (lighting, camera's ability to capture the object, and key points for measurement).

Images below illustrate some of the measurements students might make with their smartphone measuring apps:



As students work, consider some of these questions to learn more about their approach:

- *How did you decide where to stand? How did you decide what other positions to try?*
- *What have you noticed about how this app works? What impact do you think that has on what and how you measure things?*

As students share their approach with classmates, invite them to make sense of each other's work. Consider the following questions:

- *What are some things you appreciate about this approach?*
- *What questions does this approach surface for you?*
- *What can you take from this explanation to apply to your own work?*

Following the sharing and discussion, give students an opportunity to revise their work or build upon their work to share with each other again. Focus this second discussion on changes or adjustments they have made based on their peers' work.

As students return to work on the remaining prompts, monitor and select students who use similar triangles and/or trigonometry as forms of indirect measurement, as well as other mathematically promising approaches. Ensure tools for measuring angle of elevation are readily available for use or to be assembled.

As they share their approach with classmates, invite students to make sense of each other's work. Consider the following questions:

- *What are some things you appreciate about this approach?*
- *What questions does this approach surface for you?*
- *What can you take from this explanation to apply to your own work?*

Following the sharing and discussion, give students an opportunity to revise their work or build upon their work to share with each other again.

Encourage students to consider sharing their findings and suggestions for improvements with app developers. Invite students who hear back from developers to share the impact of their work with the class.

In this example, students are:

- given a cognitively demanding task. They are asked to execute a plan for determining the accuracy of smartphone measurement apps. Students also have the opportunity to use indirect measurement techniques to compare measurement results (LP 1).
- working collaboratively in pairs and sharing their findings with the class. Students are encouraged to ask questions of each other to better understand the thinking process, as well as given space to learn from their peers (LP 2).
- building conceptual understanding of the use of trigonometric ratios through indirect and actual measurement (LP 3).
- given space to develop agency, as they choose their solution pathways and reflect on how their approach might change based on engagement with their peers' work (LP 4).
- given an opportunity to see themselves as mathematicians, as their methods for generating data to support conjectures are central to this investigation (LP 5).
- using technology as a tool and learning to determine accuracy of such tools (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Creative Process:** [Innovative thinking](#) (OT.Creat.1.c)

References

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