

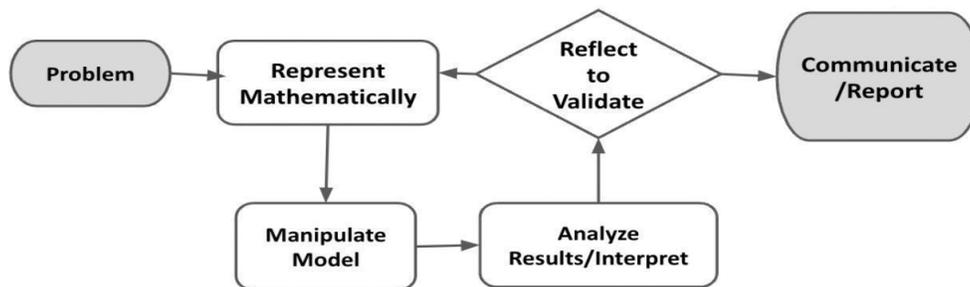
# M111B Performance Task

## Measuring Up

### The Modeling Cycle

In this task, you will examine statistics associated with a sports champion, a sleep study of Americans, or another topic of your choice to compare to your own analysis of a smaller, local population and apply the basic mathematical modeling cycle to make predictions, interpret information, and report your findings.

Mathematical models are used to make sense of ideas and phenomena in our world. As illustrated in the diagram below, the modeling cycle is a process of collecting data about an idea or problem, creating models of that data, analyzing the story told by those models, and reporting the results to the appropriate audience.



**Figure 1.** *Diagram of Modeling Process.* From “Progressions for the Common Core State Standards for Mathematics” [Draft manuscript], by CCSS Writing Team, 2019, p. 6.

In this task, you will be engaging with this mathematical modeling cycle to analyze data, evaluate findings, and identify trends. You will use the models and your takeaways from this task to create a product that reflects your interpretation of your data. You will generate and describe statistics such as standard deviation and median, create and justify a visual to explain your data, and identify an audience who would benefit from your data, among other final questions and tasks.

### Introduction: Measuring Up

We compare a lot as a society. Who is the best actor? Which car is the better buy? How much does the typical American spend on toilet paper? It is how we determine the best, worst, and typical of everything in populations of people, places, and things. Any time we collect numerical data for one variable, we are analyzing one-variable statistics.

## QUESTION 1



Think about yourself. Make a list of things you enjoy and/or interesting things that are happening in your life. Consider facts, interests, and/or talents.

*[This question is not meant to be scored. Look for reasonable responses (see teacher-facing guide for ideas). Expect students to answer individually on their own forms, as well as participate in a small group or class discussion.]*

### Student A

.Playing the horn  
.Fishing  
.Hunting  
.Dogs  
.Sundays  
.Cooking

*The list provided by the student is acceptable. This introductory question is meant to help students begin to think about things they will later try to quantify.*

### Student B

- I play football
- Going to the movies
- Riding rides
- Driving

*The list doesn't have to be long. This response is acceptable.*

### Student C

Some things I am interested in are volleyball, painting, weightlifting, etc. Some things I do outside of school is ASL online, some other things I like to do are hangout with friends, family and watch movies and play board games.

*While this response seems to be more descriptive than Student A's or Student B's response, it doesn't make it "better" than the other two responses.*

## QUESTION 2



Considering what you shared in Question 1, do you think you do more, do less, are better, or are worse? Or are you average, in the middle?

What quantities could be measured to make a comparison? Why would a comparison be important?

### Student A

I believe i known more about Horror media and movies then the average person  
I believe i do better than most people at Taekwondo  
I believe im average at reading as i read 1-2 books within a few weeks  
I believe i could do better during Latin class and studies  
I believe that i'm in the middle of knowing Greek and Roman mythology as i known more then people who don't learn about it and less then people who have taken those classes

I could measure how many horror movies I've watched in a year or how many medals I've won at taekwondo tournaments. Comparisons are important in order to see how i could improve or how well I'm doing.

*This student detailed how they compare to each of their responses in Question 1 and thoroughly answered which quantities could be measured and why a comparison is important to them.*

### Student B

I consider myself above average in those things. For football I play in every game for the gym i can lift more than a lot of people in my grade and my age. The comparison is important to see who's better and to rank people.

*This student did indicate a couple of quantities which could be measured to make a comparison and he did reference why making a comparison is important to him. Teachers don't need all responses to be as detailed as Student A's. This response is adequate.*

### Student C

I think I spend more time doing volleyball and ASL than others I know. A comparison could be how many aces you get and how many kills or digs you get. This comparison is important because it's the way the game is scored. For ASL it could be how many assignments a week I complete and the grades can show how skilled I am at ASL.

*This student is consistent in referencing their answer in Question 1 related to both volleyball and ASL. The student does offer up potential variables that can be measured (number of aces or kills or digs for volleyball and number of completed assignments for ASL).*

## Measuring Up

### Choose Your Own Adventure

In this performance assessment, you will compare statistics already compiled by a group like the U.S. Census, ESPN, or another source related to the population of your choosing. You will need to gather data from an appropriate sample of your population, compute statistical measures, build models like dot plots, and compare these to pre-prepared statistics.

---

**Your Task:** Pick one of the options below:

**OPTION A:** Compare a [famous athlete](#) to [yourself or one of the athletes in your school](#). They will need to play the same sport. You will need to select one variable to compare (e.g., compare the 3-point throws for Stephan Curry and yourself or starting point guard).

---

In the **Problem** and **Formulate** sections, you will gather data for your option, then evaluate this relationship to identify a model of best fit and create a related equation. The **Problem Formulation Checklist** can be used to ensure you have the necessary components before moving on to the remaining parts of the task.

**Note:** You are welcome to use additional paper or digital forms to document your work. Be sure to include all of your work in your final submission.

## Problem

### ACTION 1



Use search engines to help you explain why your selected comparison matters. Summarize the following information:

- State your option and what you intend to compare.
- Provide a purpose to your option.
- Describe what you expect to learn.

#### Student A

- State your option and what you intend to compare.

I am comparing high schoolers 3-point average to a famous athlete's 3-point average in high school. I think that it will be very low compared to most famous athletes' average.

- Provide a purpose to your option.

My purpose is that if some have a high 3-point average shots made per game that they are probably a higher or highest scorer in the game.

- Describe what you expect to learn.

I expect to learn how much better these famous athletes' average in high school is than a high schooler now.

*This student formulated a question that can be answered using statistical analysis and is ready to move to the next part of the task.*

**Student B**

- State your option and what you intend to compare.

I am comparing the amount of sacks I had to a professional athlete.

- Provide a purpose to your option.

The purpose of my option is to see how many more sacks I need to go D1.

- Describe what you expect to learn.

I expect to learn how many more sacks I need to go D1.

*This student formulated a question that can be answered using statistical analysis. At this point, the teacher doesn't know which professional athlete the student will be using as a comparison, but he does clearly intend to use his own stats as part of the comparison.*

**Student C**

- State your option and what you intend to compare.

I intend to compare height and how many kills and aces she has per game.

- Provide a purpose to your option.

My purpose is to see how many aces and or kills she gets per game

- Describe what you expect to learn.

See how much she gets set and serves she completes a game

*The student hasn't yet settled on a single option (number of kills, aces, sets, etc.), but is beginning to make a decision as to which activity (volleyball vs ASL) she intends to further explore in this task. While this response could be improved, the key will be whether the student is able to reduce this to a single question upon which to collect data in Action 2.*

**ACTION 2**

Write a one-variable statistical question to collect data based on your response in Action 1.



*[Teachers should spend time with each student at this point to ensure each student has a researchable question with varied and quantifiable data. Students should also be encouraged to think about what sample population they will ask. Three typical responses are shown below.]*

**Student A**

How many 3-point shots were averagely made in Lamelo Ball's high school game compared to a high schoolers game.

**Student B**

How many more sacks does a professional athlete have compared to me?

**Student C**

The amount of hits she has per game.

### ACTION 3



Define the variable you will investigate through your statistical question and the population that you will sample.

**Student A**

Variable- Average number of 3-point shots made in a high school game.

*The student identified the variable (number of 3-point basketball shots in a high school game) but didn't specify the high school. We might assume it's the student's high school, but the sample population should be specified.*

**Student B**

The amount of sacks I have.

*The student has identified the variable (number of sacks) and population (himself); however, the student should have included the comparison population here as well. It would help to ask for clarification as to which professional athlete would be used or whether the student would be looking for an average number of sacks for professional football players playing a similar position to the student.*

**Student C**

Variable- number of hits per game. My target population is anyone who plays volleyball.

*The student has adequately identified the variable (number of hits per game) and population (anyone who plays volleyball).*

## Formulate

### ACTION 4



Describe your data collection plan. Refer to these guiding questions:

- Can you investigate your statistical question? Or do you need to narrow your focus to make your question more specific?
- What data can help you investigate your statistical question?
- How much data should you collect?
- How will you collect your data?
- What type of bias might you encounter?
- How can you minimize bias?
- How frequently do you need to collect data?

**Student A**

- Can you investigate your statistical question? Or do you need to narrow your focus to make your question more specific? *I can definitely investigate my statistical question.*
- What data can help you investigate your statistical question?
- How much data should you collect? *I will collect 10 high school students, 5 boys and 5 girls*
- How will you collect your data? *I will ask 3-point shooters in high school their average per game.*
- What type of bias might you encounter? *Boys vs. Girls or 3-point shooters vs. a center*
- How can you minimize bias? *I can minimize bias by asking boys and girls who play high school basketball and shoot.*
- How frequently do you need to collect data? *Frequency doesn't really matter for my study.*

*The student answered this question specifically for the high school students and didn't address how they will collect the data for their pro player – LaMelo Ball. The student shows understanding that they need to collect data specifically from high school basketball players who "shoot," not just the general population of the school*

**Student B**

- Can you investigate your statistical question? Or do you need to narrow your focus to make your question more specific? **Yes, I can look it up on Google.**
- What data can help you investigate your statistical question? **To make sure they just count sacks.**
- How much data should you collect? **A lot**
- How will you collect your data? **Looking it up on Google**
- What type of bias might you encounter? **none**
- How can you minimize bias? **I don't have any bias**
- How frequently do you need to collect data? **At least once a week.**

*The student's answers lack specifics that are needed for the teacher to know how the data will be collected. Since this student's question is "How many more sacks does a professional athlete have compared to me?" the information provided here doesn't clearly inform how they will investigate the question. Ask questions such as "What are you looking up on Google?" or "What do you mean by "a lot"?" And the frequency seems to suggest that the number of sacks might not be specific to a game, since most high schools only allow one game per week. The teacher should direct this student to clarify their response to this question.*

**Student C**

- Can you investigate your statistical question? Or do you need to narrow your focus to make your question more specific? **Yes need to keep track of the hits or aces completed/**
- What data can help you investigate your statistical question? **A score sheet for volleyball which consists of the players number and how much they serve. For hits you can use paper and keep track.**
- How much data should you collect? **Multiple games to see her average hits and serves per game.**
- How will you collect your data? **Using score sheet and keeping stats on their hits**
- What type of bias might you encounter? **Not much bias**
- How can you minimize bias? **Not really**
- How frequently do you need to collect data? **Every game**

*The student has a plan for collecting data but needs to decide on the number of players for which she'll collect data as well as which variable she'll settle on for the remainder of the task.*

### Problem Formulation Checklist

Requirement	Fulfilled?
I clearly identified what I will be comparing and its significance.	<input type="checkbox"/>
I wrote a measurable, one-variable statistical question.	<input type="checkbox"/>
I defined my variables and sample population.	<input type="checkbox"/>
I described a clear data collection plan that addresses bias.	<input type="checkbox"/>
<b>Teacher Consultation:</b>	_____

In the **Represent Mathematically** and **Interpret & Analyze** sections, you will collect your data, compute statistical measures of center and spread, and model the data with an appropriate visual display. Write a statement explaining why you chose this particular display. The **Represent Mathematically Checklist** can be used to ensure you have the necessary components before moving on to the remaining parts of the task.

## Represent Mathematically

### ACTION 5



Carry out your data collection plan. Describe any modifications, if any, you needed when carrying out your data collection plan.

*[Most students just reported their data and didn't describe any modifications they may have needed to perform. It is acceptable to just see data at this point, as long as it aligns to their identified question in Action 2. If it doesn't align, the teacher should ask for clarification as to why they modified their data collection plan.]*

**Student A**

52, 6, 0, 8, 3, 1, 3, 100, 10, 5, 100, 500, 2, 100, 2, 11, 15

3-point average shots made a game

*The teacher should ask the student to explain the wide variety of numbers reported here. Saying this represents "3-point average shots made a game" doesn't explain how 500 could be possible. Is it for both teams? High school teams or professional teams? The student should be asked to verify the data.*

**Student B**

4, 5, 6, 6, 6, 7, 8, 2, 3, 10, 13, 3

*At this point, it is still not clear if this data represents the number of sacks the student had or a professional athlete or both. The teacher should ask the student to be more descriptive as to what these numbers represent.*

**Student C**

0,9,4,5,3,4,4,9,7,3,5,10,9,15,5,5,8,10 hits per game

*The student seems to have settled on collecting data for number of hits in a volleyball game and is no longer considering aces, digs, etc. This is an adequate response.*

## ACTION 6



Generate the following statistics for your data set.

- mean
- standard deviation
- median
- interquartile range

Explain the meaning of these statistics in the context of your data set.

*[All three students found appropriate statistics for their data sets but failed to explain the meaning of these statistics. Students B and C also neglected to determine the standard deviation for their data set. The teacher made need to emphasize that the interpretation of these statistics is part of the task. However, it is acceptable if students provide these explanations in some of the remaining actions/questions.]*

### Student A

- Mean=54
- standard deviation=112.05
- median=8
- interquartile range=73.5

### Student B

Mean $\bar{x}$	6.08333333333333
Median $\tilde{x}$	6
Mode	6
Range	11
Minimum	2
Maximum	13
Count $n$	12
Sum	73
Quartiles	Quartiles: Q <sub>1</sub> --> 3.5 Q <sub>2</sub> --> 6 Q <sub>3</sub> --> 7.5
Interquartile	4
Range IQR	
Outliers	none

### Student C

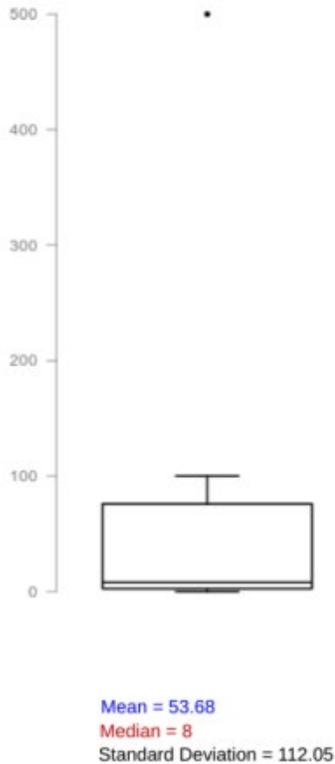
Mean $\bar{x}$	6.38888888888889
Median $\tilde{x}$	5
Mode	5
Range	15
Minimum	0
Maximum	15
Count $n$	18
Sum	115
Quartiles	Quartiles: Q <sub>1</sub> --> 4 Q <sub>2</sub> --> 5 Q <sub>3</sub> --> 9
Interquartile	5
Range IQR	

## ACTION 7



Choose and create an appropriate visual to display your data. Explain why the visual you chose is appropriate for your data.

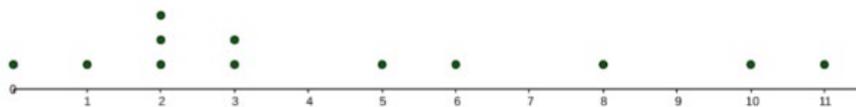
Student A



*The student question was, "How many 3-point shots were averagely made in LaMelo Ball's high school game compared to a high schoolers game?"*

*The student created two visual displays but did not explain why their visual representations are appropriate. The large outlier (500) also makes it hard to interpret the data. Additionally, the dot plot doesn't seem to present the data appropriately. The teacher should ask the student to evaluate their displays and explain how they connect meaningfully to their data.*

Mean = 53.68  
Median = 8  
Standard Deviation = 112.05



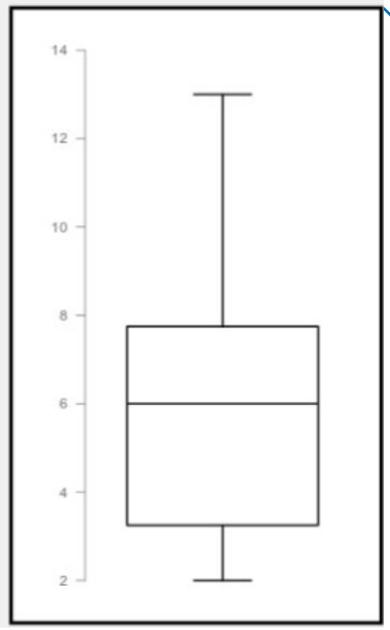
	A	B
1	52	
2	6	
3	0	
4	8	
5	3	
6	1	
7	3	
8	100	
9	10	
10	5	
11	100	
12	500	
13	2	
14	100	
15	2	

**Student B**

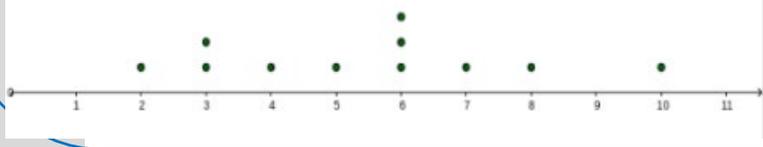
The student created two visual displays but did not explain why their visual representations are appropriate. Asking questions such as “What does this visual representation tell us about the data?”, “What are the pros and cons of each representation?”, or “What do you want your visual representation to show?” might help the student provide an adequate response. Note that the dot plot does show the standard deviation that was left out of their previous answer.

Population size: 12  
 Median: 6  
 Minimum: 2  
 Maximum: 13  
 First quartile: 3.25  
 Third quartile: 7.75  
 Interquartile Range: 4.5  
 Outliers: none

- Mean
- Geometric mean
- Harmonic mean
- Median
- Mode
- Standard Deviation
- Variance
- Mean absolute deviation
- Range
- Interquartile range
- Quartiles
- on data



Mean = 6.08  
 Median = 6  
 Standard Deviation = 3.01

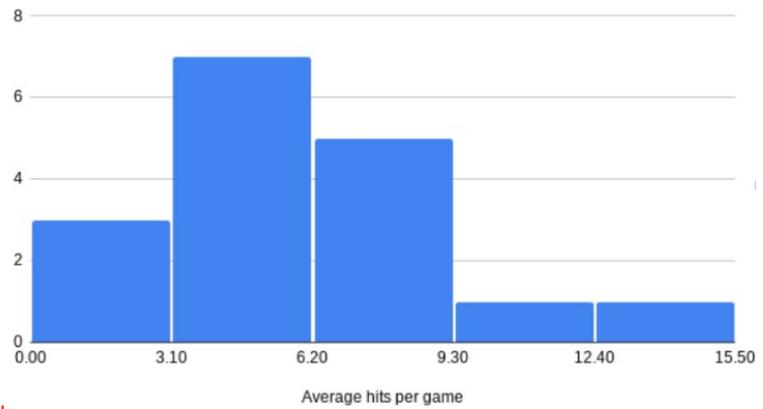


The student question was, “How many more sacks does a professional athlete have compared to me?”

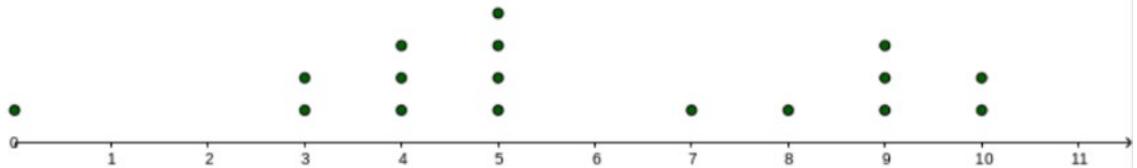
**Student C**

The student created two visual representations of their data but did not explain why their visual representations are appropriate. Asking questions such as “What does this visual representation tell us about the data?” or “What do you want your visual representation to show?” would benefit the student. Note that the dot plot does show the standard deviation that was left out of their previous answer, but it doesn't match the data as the dot plot shows no points above 10.

Histogram of Average hits per game



Mean = 6.39  
 Median = 5  
 Standard Deviation = 3.43



## Represent Mathematically Checklist

Requirement	Fulfilled?
I completed and compiled the data collection to be included in the report.	<input type="checkbox"/>
I described any modifications that may have been necessary in my data collection.	<input type="checkbox"/>
I generated the necessary statistics and interpreted each of them.	<input type="checkbox"/>
I chose a data display that is appropriate for the data set.	<input type="checkbox"/>
<b>Teacher Consultation:</b>	_____

In the **Analyze and Interpret Results** section, you will include an interpretation of your data. Use the measures you calculated above to support your response to the questions posed. The **Analyze and Interpret Results Checklist** can be used to ensure you have the necessary components before moving on to the remaining parts of the task.

## Analyze and Interpret Results

### ACTION 8



Choose the measure of central tendency and spread appropriate to your data. Explain your rationale.

*[Students should state which measure of central tendency and measure of spread is most appropriate to describe their data (i.e., mean and standard deviation or median and IQR). Students should then provide a brief explanation based on mathematical evidence to support their claim. The following student responses seem to suggest that the teacher should emphasize that a measure of central tendency and spread is specifically required in Action 8. Student B did attempt to specify a measure of central tendency, but the response needs a measure of spread and further explanation to make it adequate. Students already calculated each of these measures, however it seems there is a gap in understanding of what they mean.]*

**Student A**

I feel that the dot plot shows my data the best because It is easier to read and understand. I prefer dot plots for things like this.

**Student B**

The tendency of most of the data is 6 it is rationale because that's the number most people got when asked how many sacks they can get.

**Student C**

The best graph that would represent what I am trying to test would be the dotplot because it shows how many hit per game and shows how many times she gets the same hits per game.

## ACTION 9

Decide how you will determine and address outliers. Provide a summary of the evidence you considered when addressing outliers.

*[Here, again, these students seem to lack an understanding of how an outlier can mathematically be determined from the data. It might be beneficial for teachers to remind students how to perform the calculations to determine if outliers exist.]*

**Student A**

I will determine my outliers by seeing which one is alone and there is just one and if I graph it and there is one that is sticking out, but to my knowledge I do not have any outliers that I see.

*Encourage the student to explain how they can mathematically determine outliers from their data set. It is obvious that 500 is an outlier, and the teacher should ask the student why he thinks he doesn't have any outliers.*

**Student B**

I will exclude the outliers because they will be too high of a number or too low of a number to get a good average.

*Encourage the student to explain how they can mathematically determine outliers from their data set. While their data set may not appear to display any outliers, you still want to see evidence that they considered them and can mathematically exclude outliers.*

**Student C**

My outliers would be any drastic changes in the hitting occurrence. The outliers would be me hitting above average or below average.

*The student doesn't understand what an outlier is mathematically. It would benefit the whole class to have a discussion about how to determine outliers from a data set.*

## QUESTION 3



Recall in **Action 1** that you shared the purpose of the comparison you selected.

What story did the data tell relative to your purpose?

Who would benefit from these data, and how would they benefit?

**Student A**

What story did the data tell relative to your purpose? It told me that teenagers' average 3-point shots made a game is very low for most. Some people made quite a lot of shots so I assume that they would be the highest scorers.

Who would benefit from these data, and how would they benefit? Coaches would benefit from this data because if they pick someone with a high 3-point average they could win most games.

*The teacher should ask the student to connect their answers from the initial question they asked to this question. It's still not clear if their data included both high school students and professional players at this point. While their comment about who would benefit is valid, they still need to clarify their data collection.*

**Student B**

The data tells me that there are a lot more outliers on the top side so that tells me there is a big jump from average to great. The people that would benefit from this data is people that are trying to compare themselves to the best people

*The teacher should ask the student to explain what they mean by “a lot more outliers on the top side.” Help them understand what an outlier is mathematically as it is not clear the data supports this statement. Also, ask the student how comparing themselves to the “best people” benefits them. Do they set goals? Use this as motivation?*

**Student C**

The majority of the sample hits are between 0-15 so I can draw conclusions about how much a person hits per game and their average hits. Since the samples were directed to anyone who participates volleyball it would be the group of individuals male or female who enjoy or play volleyball.

*Again, the dot plot in the previous question is missing any points above 10, but the student is correct in stating their data collection yielding results from 0 to 15, inclusive. The teacher should ask the student how knowing how many hits they have benefits them.*

## Analyze and Interpret Results Checklist

Requirement	Fulfilled?
I selected the measures of central tendency and spread that are accurate and appropriate for the data.	<input type="checkbox"/>
I explored the possible effects of outliers using the shape, center, spread, and context of data sets.	<input type="checkbox"/>
I told the story of the data that accurately interprets and uses the results of the analysis to explain how this information can be useful.	<input type="checkbox"/>
I identified the audience who would benefit from this data.	<input type="checkbox"/>
<b>Teacher Consultation:</b>	_____

In the **Validate and Report** section, you will add on to the document you started above and clarify the voice and audience. In the **Validate** section, you will explain whether the data were sufficient for addressing your statistical question or how you would improve the process in the future. You will also create an appropriate way to report the findings from your work to the intended audience (e.g., through a short video, pamphlet, or other visual). The **Validate and Report Checklist** can be used to ensure you have the necessary components for completing the task.

## Validate and Report

### Action 10



Validate your model using these guiding questions.

- Did the data sufficiently address the question you were investigating? Why or why not?
- If you were to try to collect data to answer your statistical question again, how would you adjust your process, your question, and/or how you analyzed the data to provide better information to the audience interacting with your data?

#### Student A

- Did the data sufficiently address the question you were investigating? Why or why not? **No, because some people were probably lying about their average shots made.**
- If you were to try to collect data to answer your statistical question again, how would you adjust your process, your question, and/or how you analyzed the data to provide better information to the audience interacting with your data? **I would ask the people I'm asking to show me proof instead of just taking what they say, so I could back of my claim**

*The student adequately answered the questions. However, the answers were short and did not show much reflection even though the student did finally address why some of their data seemed so unreliable (i.e., 500, 100, etc.). The teacher should ask the student whether the information they were seeking could have been independently reported, for example via MaxPreps. Also, the student never did show the data they intended to compare against (LaMelo Ball's high school stats).*

#### Student B

The data didn't address the question because I chose some numbers from professionals and some from my classmates and I didn't choose enough from the professionals. If I were to try this again I would tweak my question and say what is the average number of sacks a student could get compared to the average number of sacks a professional could get.

*The student seems to have recognized the weakness in their initial data collection. This kind of insight is part of what this task intends to elicit from a student.*

**Student C**

My data does address my question about how much players hit per game. If we were to collect data again I would probably collect more data by collecting more evidence and more data about how much people hit per game.

*The student seems to have recognized that her data set was small and could be expanded. The teacher could ask her what her target population would be if she were to start again.*

### ACTION 11



Communicate your results in a manner and format that are appropriate for the audience and purpose you identified in Questions 2 and 3, in a way that most clearly illustrates your findings.

*[Teachers should be looking for responses that are supported with the mathematical evidence calculated and represented. Teachers can support students by revisiting Action 6 and reviewing the statistical analysis, ensuring that students were able to give contextual meaning to their calculations. Three typical (but inadequate) responses are shown below.]*

**Student A**

The people I asked said low or actual average numbers but some people said very high numbers that weren't possible in a game. Most high schoolers' average compared to Lamelo Ball's high school games were less than him.

**Student B**

The average number of sacks I would need to get is 6.

**Student C**

The information I had gathered for this experiment was very relevant to what I was studying because I wanted to know how many hits per game players got. So I got 18 pieces of information to start my work. The numbers ranged from 0-15 with the median being 5.

### Validate and Report Checklist

Requirement	Fulfilled?
I provided a rationale showing how the data sufficiently addressed the question at hand or I articulated a clear understanding about how to improve the data collection and analysis in the future.	<input type="checkbox"/>
I clearly communicated the results in a way that is appropriate for the intended audience.	<input type="checkbox"/>
<b>Teacher Consultation:</b>	_____