

# Annotated Examples: M201 Function Concepts

The examples that follow are intended to illustrate how the learning principles are used to support students' engagement with the content and practices outlined in this badge. These examples do not provide comprehensive coverage of those expectations, but rather elevate some of the learning principles that are less likely to be part of published curricular materials for mathematics instruction. The examples that follow were developed by the Math Badging writing team, unless otherwise specified. These are a small sample of types of learning activities that can be done with students, both in and out of a traditional classroom setting.

## Example 1: Way Too Fast

Students are presented with a scenario like this:

### Part A: Speeding in Massachusetts

A speeding ticket fine in Massachusetts is \$100 for any speed up to 10 miles per hour above the posted speed limit. The fine increases by \$10 for each additional mile per hour over the speed limit after the first 10.

[Source: [Speeding Ticket FAQs | Massachusetts Speeding Ticket Lawyer](#)]

1. Create a graph of this situation where the number of miles per hour over the posted speed limit is on the horizontal axis and the cost of the ticket is on the vertical axis.
  - a. Does this graph represent a function?
  - b. What domain and range are reasonable for this function? How do you see these in the graph?
2. Suppose the posted speed limit is 65 miles per hour. Create an equation that takes the driver's speed at the time of incident in that zone and generates the cost of the driver's speeding ticket. Explain how the equation you created works. What is the input? What is the output?

### Part B: Speeding in New York

Fines for speeding vary from state to state. In New York, speeding penalties are determined as follows:

Speed	Minimum fine	Maximum fine	Possible prison time
up to 10 mph over	\$45	\$150	not more than 15 days
more than 10 mph over - less than 30 mph over	\$90	\$300	not more than 30 days
more than 30 mph over	\$180	\$600	not more than 30 days
inappropriate speed	\$45	\$150	not more than 15 days

Source: [Penalties for Speeding | Governor's Traffic Safety Committee](#)

1. Create a graph of this situation that illustrates the relationship between quantities.
  - a. What is interesting about your graph?
  - b. What do some of the interesting features of the graph mean relative to the context?
  
2. How does New York's Speeding Penalty system compare to Massachusetts' system?
  - a. What are your thoughts about this?

### **Part C: Extending the Learning**

Learn about your state's or another state's system for determining speeding penalties.

- Is it possible to represent the system graphically? Using an equation of some sort? Why or why not?
- How does your state's or the other state's system compare to the systems in New York and Massachusetts?
- What are your thoughts on this?

### **Sample Learning Activity**

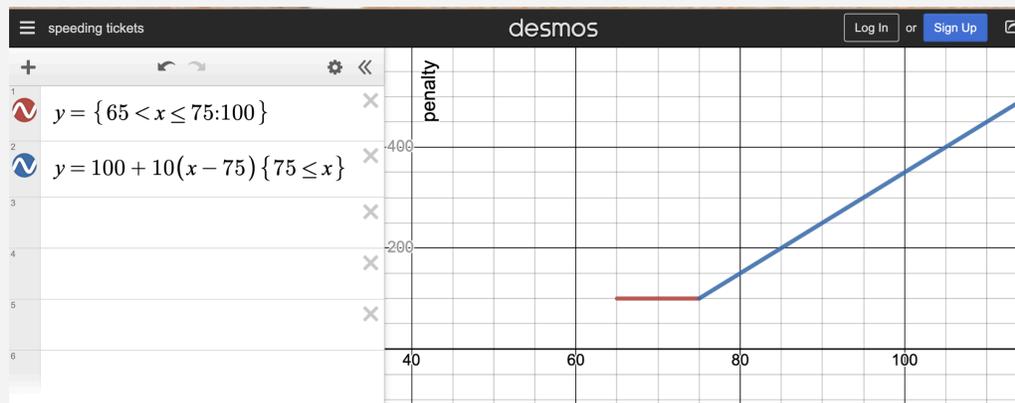
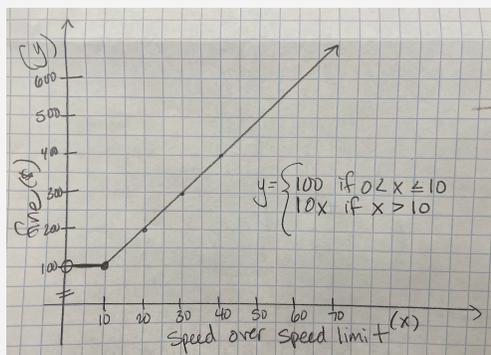
Launch this task by engaging students in a conversation to make sense of driving laws, speed limits, and penalties for going over the speed limit. Some questions you can pose are as follows:

- How does a driver determine how fast to go at any point in time?
- How does a driver know the maximum speed for any road they're on?
- What are some consequences for going over the speed limit?

Introduce the prompt and provide time for students to consider the questions posed. Give students time to work independently and develop a response. Ensure students have access to multiple tools they can use to develop a response.

Invite students to partner with classmates and share their responses. Provide time for students to revise their justifications using the information gained during these discussions.

Some possible approaches are as follows:



Monitor and select various solution methods to share with the class as students work with classmates. Have selected students present their responses—consider using a document camera for students to show actual work—and explain their justification. As discussion takes place, invite classmates to make sense of and build on each other’s responses, allowing for students to clarify and defend their response as the conversation ensues. Ask:

- *What about this reasoning makes sense?*
- *In what ways is using a function helpful to communicate speeding penalties?*
- *What questions surface for you?*
- *Is there anything you have as an alternate explanation or point of view?*
- *What can you take from this approach to apply to your own work?*

Following the discussion, provide students an opportunity to reflect on and revise their work. Additionally, offer students tools to explore any of the questions that surface from examining the speeding penalty practices of these states. Follow a similar structure for Parts B and C.

In this example, students are:

- given a cognitively demanding task. Students grapple with determining which quantities work together to determine a function grounded in a context. Students have multiple entry points to reason about this prompt in various ways (LP 1).
- using the varied structures that give opportunities to determine an approach by working independently, then sharing with a partner or class, then revising their response based on those interactions (LP 2).

- making sense of another person’s line of mathematical reasoning as well as critiquing the mathematical reasoning that surfaces (LP 3).
- given the opportunity to develop their mathematical identity since their ideas and reasoning are the foundation of the discussion (LP 4).
- given the opportunity to engage in mathematics that authentically involves a real-world situation (LP 6).
- considering how a context can be represented using mathematical tools, as well as seeing how the idea of a function relates to the idea of a just system. In doing so, students have an opportunity to broaden their ideas about what mathematics is, how it is used, and who it is for (LP 5).
- using technology to represent this context and show and explain connections between an equation and a graph (LP 7).

Through the implementation of this learning activity, look for opportunities to support students’ growth and development on this XQ competency:



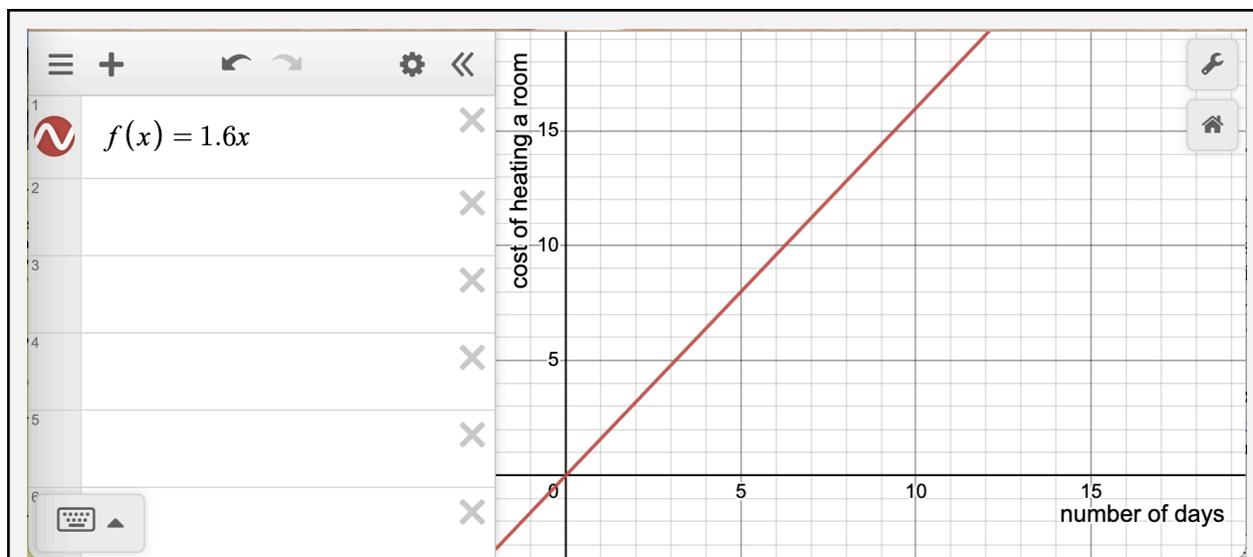
- **Understand Governments:** [Public policy](#) (FK.SS.1.c)

### Example 2: Turn Up the Heat

Students are given a problem like this:

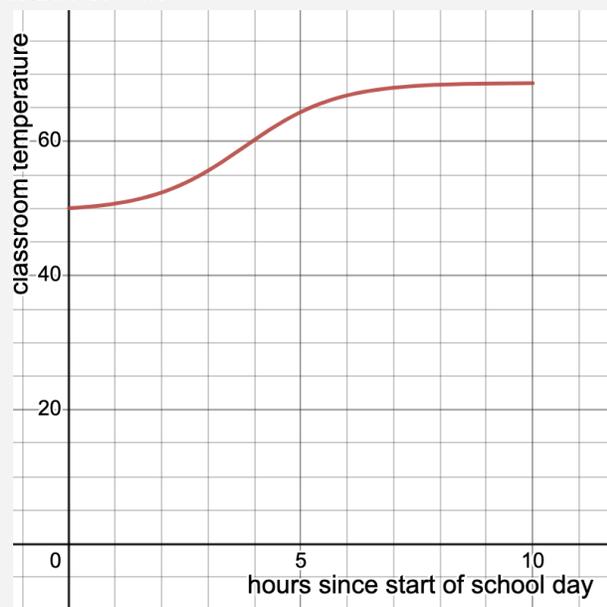
*During the winter months, some schools shut down due to extreme cold conditions and the schools’ inability to adequately heat their buildings. According to the Capital Gazette and Bloomberg, the Baltimore City Public School System experienced this over the course of two winters in 2017 and 2018. In an effort to help, one college student started a [GoFundMe campaign](#) to purchase space heaters, raising about \$84,000. One high school student reported near freezing temperatures of  $50^{\circ}\text{F}$  at the start of the day. With the use of a space heater, the classroom temperatures increased to  $65^{\circ}\text{F}$ . In Maryland, the estimated cost to run a heater is about \$0.20 per hour.*

- Examine the image below or explore the actual graph at <https://www.desmos.com/calculator/nts1vbqygf>
  - How would you describe the relationship between the input and the output?
  - What does this relationship tell you about the costs of heating a room? How do the variables, slope of the line, and the equation of the function capture that relationship?
  - If a school has set up 40 classrooms each with a space heater, how would the function between the number of days and the cost of heating change? How does this impact the graph of the function? How does it impact the equation?



b. Consider the following graph of  $y = g(x)$ .

- i. How would you describe the relationship between the input and the output?
- ii. What values of the input are part of the domain of this function? Why does that make sense?
- iii. What does the relationship represented below tell us about how a space heater works to warm a room?



- iv. Given the conditions, the school leadership team decides to have administrators who are already in the building two hours before the start time preheat all the classrooms and keep the heaters running until the end of the school day.
  1. Sketch a graph of the new function  $h$ , which gives the temperature in the school as a function of time.
  2. Write an equation for  $h$  in terms of  $g$ . Explain why your equation makes sense.

3. *How will the school leadership team's decision to start heating rooms earlier impact their cost for heating the school?*

### Sample Learning Activity

Have students examine the following articles to set context to the situation. Lead a class discussion based on thoughts, reflections, surprises, etc. from the articles.

- [How Baltimore's Public Schools Got Left In the Cold - Bloomberg](#)
- [Baltimore closes public schools after students sat in cold classrooms | CNN](#)

Lead into the activity with some individual work time, where students develop an approach to the prompt, encouraging them to share and compare approaches. Monitor and select a couple of approaches to highlight and reflect on progress made at this point.

Questions that can be posed to student pairs as they work:

- *What is this context about? What connections can you make to your own experience?*
- *What are the quantities—things that can be counted or measured—involved in this context?*
- *What do you anticipate is the relationship between these quantities? Why does that make sense?*
- *What connections can you make between these ideas you shared and the first graph?*
- *What connections can you make between these ideas you shared and the first equation?*
- *Press for details by asking can you say more about...?*

Give selected pairs of students time to share responses and invite classmates to reflect on the following questions:

- *What makes sense about the ideas shared?*
- *What questions surface for you?*
- *What can you take from this discussion and apply to your own work?*

Provide time for students to revise their work. While they work, select additional pairs to share how they used the structure and relationship between the equations to determine which sets of equations have the same answer. Give students additional time to revise their work.

Additionally, offer students tools to explore any of the questions that surface from examining the case of Baltimore Public Schools.

In this example, students are:

- given a cognitively demanding task. They are asked to analyze the parameters of a scenario in order to apply it to a linear function. Students have multiple entry points to reason about the functions in play by way of multiple representations and work to describe implications of changes in parameters and considerations on the function (LP 1).

- working collaboratively in pairs to complete the task, allowing for social interaction with peers. Students are also encouraged to ask questions to their peers to understand each other’s thinking processes and revise their own thinking (LP 2).
- making sense of multiple representations and how they relate to each other—verbal, algebraic, numerical, graphical. Additionally, students work to communicate correspondences between the various representations, deepening their understanding of the concept of a function (LP 3).
- encouraged to develop skills of expert learners, thereby developing agency, as they reflect on their approach to the prompt and incorporate new ways of thinking about this from their peers (LP 4).
- considering how a context can be represented using mathematical tools, as well as how changes in the context impact these representations. In doing so, students have an opportunity to broaden their ideas about what mathematics is, how it is used, and who it is for (LP 5).
- given the opportunity to engage in mathematics that authentically involves a real-world situation (LP 6).

Through the implementation of this learning activity, look for opportunities to support students’ growth and development on this XQ competency:



- **Economics:** [Economic forces](#) (FK.SS.3.b)

### Example 3: You Are a Winner!

Students are given a situation like this:

*Congratulations! You have just won the lottery! You are given a choice of how to receive the money. Would you rather receive a lump sum of \$1,000,000 or annual payments spread out over several years?*

- Without doing any calculations, which is your initial choice and why?*

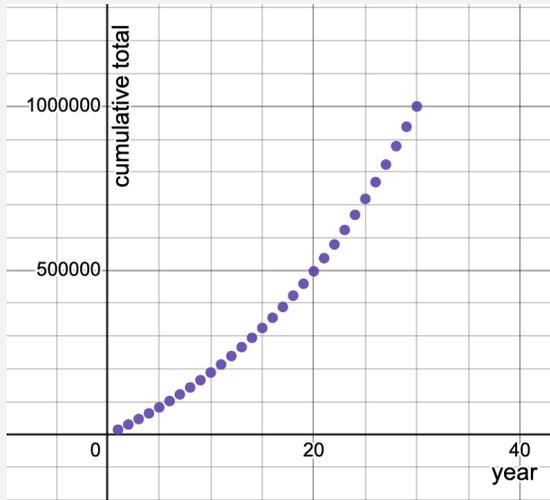
*Let’s learn more about how lotteries work and the options offered. Take a moment to read Jaime Johnson’s article “How lotteries work and 3 key steps to take if you win one” published November 1, 2022 on Business Insider. [[How lotteries work and 3 key steps to take if you win one](#)]*

- If you choose the lump sum option and place the money in a savings account, we can use a function to represent the amount of money in the account after  $t$  years.*

*One such function is  $f(t) = 538200 \cdot 1.0018^t$*

- Based on what you learned from the article, explain what each of the parameters (variables and constants) in the function represent.*
- What type of function is this? Why does it make sense to have this type of function in this case?*
- The option of having annual payments over time is referred to as an annuity. If you choose a lottery annuity, you agree to receive an initial payment followed by yearly payments. In the case of the Mega Millions lottery, winners agree to receive 29 yearly payments, where each payment grows by 5% from the previous year. The image below shows the cumulative total a winner of a \$1,000,000 prize would have over the course of that period. You can also access the graph at this link: <https://www.desmos.com/calculator/f6zy9mijdh>.*

- i. How would you describe the function that fits the annuity plan?
- ii. Use the link provided or a Lottery Annuity Calculator, like [Lottery Annuity Calculator](#), to explore and identify a function that describes how the annuity program works.
  1. What do each of the components of your function mean in terms of the context?



- d. Which lottery choice is more lucrative? How do the insights gained from looking at the various mathematical representations help support your response?

### Sample Learning Activity

Give students an opportunity to independently begin exploration, then invite them to work in pairs to engage in the exploration. As students work, monitor for the different ways they communicate their reasoning.

Select pairs of students to share responses. As students share their ideas, consider use of the following questions to facilitate the discussion:

- *What are you attending to as you consider how each option works? How do the various mathematical representations make that visible?*
- *If we compare the two options over shorter time frames, like two or three years, or 12 and 20 years, what do you notice? Is that enough information to make a decision about which option is more lucrative?*
- *How do each of the options grow?*

Invite classmates to reflect using the following questions:

- *What makes sense about the ideas shared?*
- *What questions surface for you?*
- *What can you take from this discussion and apply to your own work?*

Provide time for students to revise their work. While they work, select additional pairs to share how they used the structure of the various representations, the relationship between the variables involved, and additional considerations they took into account in order to determine which option is the most lucrative. Give students additional time to revise their work.

In this example, students are:

- given a cognitively demanding task. They are asked to explore and explain how the various payout options are offered to lottery winners. Within this context, students have an opportunity to make sense of exponential functions and geometric sequences and series (LP 1).
- working individually and collaboratively as they make progress through this exploration. Students are encouraged to ask questions to their peers to understand one another's thinking processes (LP 2).
- given a task that centers reasoning with multiple opportunities to make sense of payout options as well as make connections between the underlying mathematical principles in play and their corresponding mathematical representations (LP 3).
- given the opportunity to develop their mathematical identity since their ideas and reasoning are the foundation of the discussion (LP 4).
- given the opportunity to engage in mathematics that authentically involves a real-world situation (LP 6).
- considering how a context can be represented using mathematical tools, as well as seeing how the functions can be used to make informed financial decisions. In doing so, students have an opportunity to broaden their ideas about what mathematics is, how it is used, and who it is for (LP 5).
- using technology to represent this context and show and explain connections between an equation and a graph (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Computational Thinking:** [Using computational tools](#) (FK.SS.1.c)

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