

M112: Modeling with Data- Two-Variable Measurement Data

[Illustrative Mathematics–Algebra 1 Unit 3: Two-variable Statistics](#)

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4)**. This can be accomplished by learning about students’ interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum’s mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- As the third unit, this IM is designed to follow work on linear functions. As a result, a large majority of the tasks focus on bivariate numerical data that is linear in nature. Because of this structure, it is expected that prompts early in a student’s journey through this unit give students opportunities to attend to [CPE 112.d](#). As work in the unit progresses, students should gain comfort and proficiency in using tools to fit a model to two-variable measurement data, [CPE 112.e](#). On occasion, students will have opportunities to recognize nonlinear trends. Some examples have been found to look exponential in nature. From work in this unit, it is sufficient for students to name such trends as nonlinear to meet the expectations of [CPE 112.e](#). It is encouraged that students explore regression tools using technology available to determine the choice of linear or nonlinear function to model two-variable measurement data.
- While most of the modeling cycle is well represented, there are few opportunities for students to formulate their own questions or to identify the variables of interest in a given situation or data set. The use of [Math Language Routine 5: Co-Craft Questions](#) may be useful in building these opportunities into some of the existing modeling tasks within the unit,

particularly some of the more complex tasks toward the end of the unit. This is necessary to fully meet Content and Practice Expectation 112a.

- Educators will need to bring in space to attend to Indicator 112.f.iii and Indicator 112.h.ii. Consider using this template to offer students space to reflect and document their thinking at key points in time. Places where we see an opportunity for this are indicated by a triangle ▲.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one’s thinking has excellent value in solidifying one’s knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students’ identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see

themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you...
112.a: Engage in the modeling cycle.	i. engage with the full modeling cycle (problem, formulate, compute, interpret, validate, revise as necessary, report).
112.b: Use scatter plots to represent two-variable measurement data.	i. construct (by hand or using technology) a scatter plot from collected data.
	ii. explain the meaning of a data point plotted on a scatter plot.
112.c: Describe visible patterns between two data sets in a scatter plot.	i. identify the type of association visible in a scatter plot.
	ii. give a hypothesis that might explain the association identified in a scatter plot.

Content and Practice Expectations	Indicators Choose an artifact where you...
112.d: Informally fit a straight line for scatter plots that suggest a linear association.	i. sketch a line that follows a pattern of linear association.
	ii. justify location and direction of a line fitted to data.
112.e: Fit a linear, quadratic, or exponential function to two-variable measurement data.	i. graph a linear, quadratic, or exponential function that models two-variable measurement data.
	ii. find the equation for a linear, quadratic, or exponential function that models two-variable measurement data.
	iii. justify the choice of linear, quadratic, or exponential function to model two-variable measurement data.
112.f: Use functions that have been fitted to two-variable measurement data to answer questions about the relationship being modeled.	i. explain the meaning of parameters for a function fitted to two-variable measurement data.
	ii. use a function fitted to two-variable measurement data to predict values.
	iii. describe any limitations to the model relative to answering particular questions of interest.
112.g: Assess the fit of a function using the correlation coefficient, residuals, and other tools.	i. find the correlation coefficient for a function fitted to two-variable measurement data.
	ii. explain the meaning of a correlation coefficient in the context of the problem.
	iii. plot residuals for a function fitted to two-variable measurement data.
	iv. interpret a residual plot in terms of a function's goodness of fit.
112.h: Explain insights gained from analyzing two-variable measurement data and limitations of curves fitted to data.	i. develop a proposal or recommendation grounded in your analysis.
	ii. explain a limitation of a curve fitted to data.
	iii. distinguish between correlation and causation.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	MT
112.a.i										X	X
112.b.i				X	X	X				X	X
112.b.ii				X							X
112.c.i							X	X	X	X	X
112.c.ii								X	X	X	X
112.d.i				X	X					X	
112.d.ii				X	X						
112.e.i				X			X	X		X	X
112.e.ii							X			X	X
112.e.iii							X			X	X
112.f.i				X	X			X	X	X	X
112.f.ii				X	X			X		X	X
112.f.iii								▲	▲	▲	▲
112.g.i							X	X		X	X

112.g.ii								X		X	X
112.g.iii						X				X	X
112.g.iv						X				X	X
112.h.i									X	X	X
112.h.ii								▲	▲	▲	▲
112.h.iii									X	X	X

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.1	D Focus of the lesson is not within the scope of this badge. Not needed for the next lesson.			Context: Office supplies, fruit flies, running a dentist office Math focus: Two-way tables
3.2	D Focus of the lesson is not within the scope of this badge. Not needed for next lesson.			Context: College degrees, cat population, treatment of colds Math focus: Relative frequency tables
3.3	D Focus of the lesson is not within the scope of this badge. Not needed for next lesson.			Context: Dessert preference, nitrate levels in coral reef, student choice Math focus: Associations in categorical data
3.4	M Please note this lesson is also included in Badge M102. Consider revisiting parts of this lesson, like Activity 4.1 and/or Activity 4.3, to	112.b.i (data is given) 112.b.ii 112.d.i 112.d.ii 112.f.i 112.f.ii	LP2, LP3, LP7	Context: Oranges, ingredient cost, sale prices, mass, fuel economy Math focus: Creating scatter plot from given data, identifying line of best fit, slope, y-intercept, making predictions

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	reintroduce this topic when addressing this Badge.			
3.5	K This is a review of topics from Lesson 4. Consider using this as a way to reintroduce this topic of study if you use Lesson 4 with Badge M102.	112.b.i (data is given) 112.d.i 112.d.ii 112.f.i 112.f.ii	LP2, LP7	Context: Weight of ice cream, degrees Celsius Math focus: Creating scatter plot from given data, identifying line of best fit, slope, y-intercept
3.6	K	112.b.i (data is given) 112.g.iii 112.g.iv	LP2, LP7	Context: Number and weight of oranges Math focus: Residuals, fit a function to data
3.7	M Include “Are you ready for more?” activity found in Activity 7.3 to meet CPE 112.e.	112.c.i 112.e.i 112.e.ii 112.e.iii 112.g.i	LP1, LP2, LP3, LP7	Context: Card sort, walking, texting Math focus: Linear and nonlinear models, correlation coefficient
3.8	K Activity 8.2 lends itself to the use of Form for Aligning Lessons	112.c.i 112.c.ii 112.e.i 112.f.i	LP1, LP2, LP6, LP7	Context: Temperature, free throws, student height, waiting room, cereal, distance, time, cars, trips, dolphins

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	<u>Reflection- Limitation of Curve Fitting.</u>	112.f.ii 112.f.iii 112.g.i 112.g.ii 112.h.ii		Math focus: Type of association, strength of correlation, using an equation to make predictions, interpreting the coefficients, describing variables that influence the situation, slope, y-intercept
3.9	K Activity 9.2 lends itself to the use of <u>Form for Aligning Lessons Reflection- Limitation of Curve Fitting</u> to meet 112.f.iii. Activity 9.3 lends itself to the use of <u>Form for Aligning Lessons Reflection- Limitation of Curve Fitting</u> to meet 112.f.iii and 112.h.iii. Leverage Activity 9.3 to give students access to full scope of CPE 112.h.	112.c.i 112.c.ii 112.f.i 112.f.iii 112.h.i 112.h.ii 112.h.iii	LP2, LP3, LP4, LP6	Context: Car (sale price, cup holders, oil changes, miles driven), inches of rain, rain jackets, books, holiday party, height, score of vocabulary, advertisements Math focus: Type of relationships between variables, casual relationships, description of how variables are related, correlation, causation
3.10	K Both activities in this lesson lend themselves to the use of <u>Form for</u>	112.a.i 112.b.i 112.c.i 112.c.ii	LP1, LP2, LP3, LP4, LP5, LP6, LP7	Context: Anthropologist, fossilized humerus bones, number of penalties a team receives, number of wins

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	<u>Aligning Lessons Reflection- Limitation of Curve Fitting</u> to meet 112.f.iii and 112.h.iii.	112.d.i 112.d.ii 112.e.i 112.e.ii 112.e.iii 112.f.i 112.f.ii 112.f.iii 112.g.i 112.g.ii 112.g.iii 112.g.iv 112.h.i 112.h.ii 112.h.iii		Math focus: Collecting, summarizing, interpreting, and drawing conclusions from bivariate data, best fit lines, residuals, correlation coefficients
Modeling Task- <u>College Characteristics</u>	K Students may use qualitative data to analyze, in which case lessons 1-3 of this unit will be key. In these cases, encourage students to include bivariate numerical data, which are visualized using scatter plots and	112.a 112.b 112.c 112.e 112.f 112.g 112.h	LP4, LP6, LP7	Context: College characteristics, SAT scores, ownership, acceptance rate, average cost, enrolled students, median earnings 10 years after graduation, highest degree awarded, completion rate, median ACT score Math focus: Conjecture, how data are related, presentation of data

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	lines of best fit, in order to get to the Content and Practice Expectations of this Badge.			