

Baseline Professional Learning Handout

M205 Modeling with Trigonometric Functions

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High School - Trigonometric Functions

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. ★

[High School, Functions Excerpt](#)

Extend the domain of trigonometric functions using the unit circle

After study of trigonometric ratios in right triangles, students expand the types of angles considered. Students learn, by similarity, that the radian measure of an angle can be defined as the quotient of arc length to radius.^{G-C.5} As a quotient of two lengths, therefore, radian measure is “dimensionless.” That is why the “unit” is often omitted when measuring angles in radians. In calculus, the benefits of radian measure become plentiful, leading, for example, to simple formulas for derivatives and integrals of trigonometric functions.

Model periodic phenomena with trigonometric functions

Now that students are equipped with trigonometric functions, they can model some periodic phenomena that occur in the real world. For students who do not continue into advanced mathematics, this is the culmination of their study of trigonometric functions. The tangent function is not often useful for modeling periodic phenomena because $\tan x$ is undefined for $x = \frac{\pi}{2} + k\pi$, where k is an integer. Because the graphs of sine and cosine have the same shape (each is a horizontal translation of the other), either suffices to model simple periodic phenomena.^{F-TF.5} A function is described as *sinusoidal* or is called a *sinusoid* if it has the same shape as the sine graph, i.e. has the form $f(t) = A + B\sin(Ct + D)$. Many real-world phenomena can be approximated by sinusoids, including sound waves, oscillation on a spring, the motion of a pendulum, tides, and phases of the moon. Some students will learn in college that sinusoids are used as building blocks to approximate any periodic waveform.

M205 Content and Practice Expectations

205.a	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
205.b	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers.
205.c	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
205.d	Analyze trigonometric functions using different representations.
205.e	Use trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

M205 Content and Practice Expectations and Indicators

Content and Practice Expectations	Indicators Choose an artifact where you...
205.a: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	i. use the definition of a radian to explain radian measure
	ii. explain what an angle given with a measure in degrees would be in radians
205.b: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers.	i. explain how radians lead to a dimensionless number that makes it possible to extend the domain of trig functions
	ii. use the unit circle to justify the value of sine, cosine, or tangent for an angle that is greater than 90 degrees
205.c: Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	i. describe the symmetry of a trigonometric function
	ii. use the unit circle to explain why a trigonometric function has symmetry
	iii. describe the periodicity of a trigonometric function
	iv. use the unit circle to explain why a trigonometric function is periodic
205.d: Analyze trigonometric functions using different representations.	i. create the graph of a trigonometric function
	ii. identify key features of the graph of a trigonometric function, such as its period, amplitude, and frequency and connect these features back to the context of the problem
	iii. explain the relationship between the parameters of a trigonometric function and its graph
205.e: Use trigonometric functions to	i. create a trigonometric function to model a real-world situation

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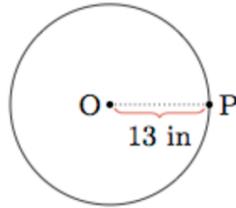
model periodic phenomena with specified amplitude, frequency, and midline.

ii. use a trigonometric function to answer a question about a real-world situation

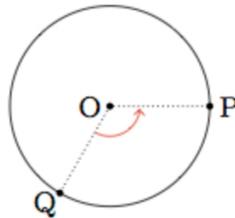
Let's Investigate

Bicycle Wheel Task

The wheels on a bicycle have a radius of 13 inches. One wheel is pictured below:



The point O is the center of the wheel and a point P on the circle is chosen. After the wheel has moved forward a distance of d , the point P moves to a new point on the circle, marked Q below:

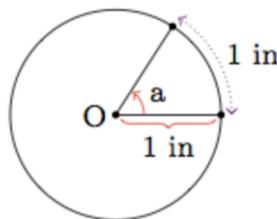


- a. Complete the table below, showing how the angle QOP varies with the distance d that the wheel has traveled forward:

Angle measurements in degrees

d	$m(\angle QOP)$ in degrees
1 inch	
10 inches	
3 feet	
7 feet	

- b. Recall that one radian is the measure of the angle cut out by a one unit length arc on a circle with radius one unit so $m(a) = 1$ radian in the picture below:



More generally, a counterclockwise angle of r radians is the angle cut out by an arc of length r units on a unit circle. Using this definition of radian angle measure, complete the table

below, showing how the angle QOP varies with the distance d that the wheel has traveled forward:

Angle measurements in radians

d	$m(\angle QOP)$ in radians
1 inch	
10 inches	
3 feet	
7 feet	

- c. What is the relationship between the measure of an angle in degrees and its measure in radians?

Source: [Illustrative Mathematics](#)

Foxes and Rabbits 2 Task

Given below is a table that gives the populations of foxes and rabbits in a national park over a 12 month period. Note that each value of t corresponds to the beginning of the month and $t = 0$ corresponds to the beginning of January.

t , month	r , number of rabbits	f , number of foxes
0	1000	150
1	750	143
2	567	125
3	500	100
4	567	75
5	750	57
6	1000	50

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7	1250	57
8	1433	75
9	1500	100
10	1433	125
11	1250	143

Note that the number of rabbits and the number of foxes are both functions of time.

- Explain why it is appropriate to model the number of rabbits and foxes as trigonometric functions of time.
- Find an appropriate trigonometric function that models the number of rabbits, $r(t)$, as a function of time, t , in months.
- Find an appropriate trigonometric function that models the number of foxes, $f(t)$, as a function of time, t , in months.
- Graph both functions and give one possible explanation why one function seems to “chase” the other function.

Source: [Illustrative Mathematics](#)

Let's do some math! Round 1

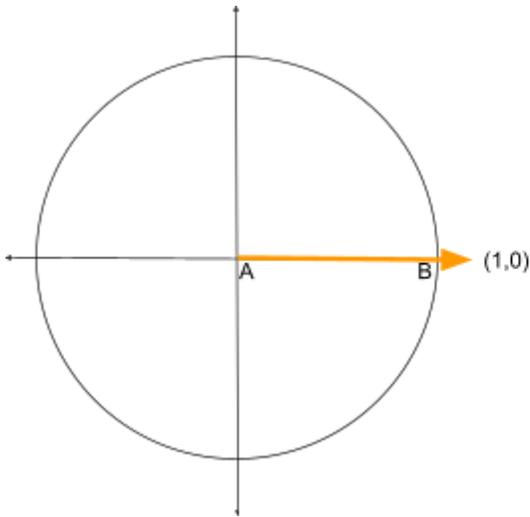
Task 1: Arc Length and Radians

Students are given a problem like this:

Part 1

Draw a circle on a coordinate plane centered at $(0,0)$ with a radius of 1. This is called a unit circle.

What is the circumference of the circle?



What is the length of the arc from point $(1,0)$ to the point $(-1,0)$? How do you know?

Part 2

Use the following steps to find additional arc lengths, angles, and coordinates on the unit circle.

- 1) Find the arc length from point $(1,0)$ to a different point on the unit circle.
- 2) Mark and label the point J.
- 3) Record the arc length in the table.
- 4) Draw ray AJ.
- 5) Determine the measure of the angle formed by ray AB and ray AJ. Record this in the table.
- 6) What are the coordinates of point J? Record the coordinates in the table.
- 7) Repeat steps 1-5 for 3 more points. Label them K, L and M.

Point	Arc Length from $(1,0)$	Angle Measure	Coordinates
J			
K			
L			
M			

Part 3

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Definition of one radian: An angle of 1 radian is defined to be the angle, in the counterclockwise direction, at the center of a unit circle which spans an arc of length 1.

Given the definition of 1 radian, approximately where would one radian be on your circle? Mark a point there and label it point P.

If you haven't already, explore the relationship between the sin and cosine of the angle measures from your table and the coordinates of the associated points. What do you notice? How can you explain the relationship that exists, if any?

Sample Learning Experience

The purpose of this task is to explicitly connect arc lengths on the unit circle to the radian as a unit of measure.

Begin by giving students quiet time to make sense of the information given and process Part 1 of the task they have to complete. Provide them with physical tools, such as graph paper, rulers, or digital geometry software to experiment with relationships made visible by the unit circle.

After some individual time, encourage students to discuss progress with a partner or small group. Encourage students to make their thinking visible. Initially, focus a conversation on how students interpreted and used the given information. Consider pulling the class together to discuss and share students' labels and use of tools to build shared understanding of the known information about the triangles. Monitor and select student work that aids in the conversation.

Ensure that students' understanding of Part 1 is secured before having them explore Part 2. Monitor for, select, and share student work that embodies different approaches and levels of formality for discussion. Listen for the various ways that students determine how to complete different columns in their table in Part 2. Did they use trigonometry or algebra knowledge to determine the coordinates? Did they estimate the angles and arc lengths or are they precisely determined?

As students share their understanding and reasoning with classmates, invite them to make sense of each other's work. Consider the following questions:

- What are some things you appreciate about this explanation?
- What questions does this explanation surface for you?
- What can you take from this explanation to apply to your own work?

Following the sharing and discussion, give students an opportunity to revise their work and share with each other again. Focus this second discussion on changes or adjustments that they have made based on their peers' work.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one’s thinking has excellent value in solidifying one’s knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students’ identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Task 1: Arc Length and Radians Student Work

Example 1:

Insert image of student work here

Example 2:

Insert image of student work here

Example 3:

Insert image of student work here

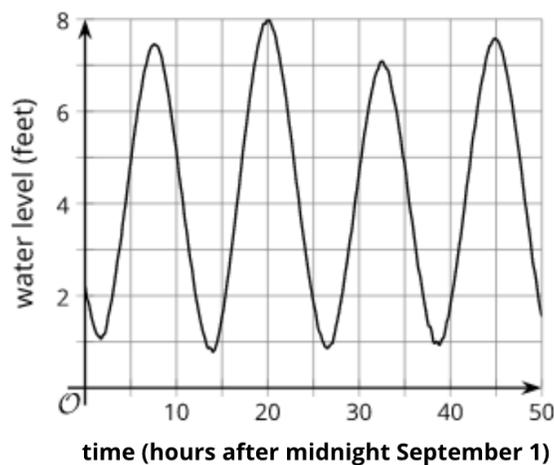
Let's do some math! Round 2

Task 2: Modeling Tides

(Adapted from IM Alg 2 6.19 Summary)

Students are given a problem like this:

Have you ever gone to the beach on a summer day and seen a yellow flag on the shore? This flag is an indication of moderate surf signaling that the tides may be high and swimmers should be cautious when entering the water. Oceanographers are people who research and study the different conditions of the ocean. One important job of oceanographers is to study the waves, currents, and tides. These reports on ocean activity help us to know when the ocean is safe and allowed to be open to the public for activities such as fishing, boating, and swimming.



1. Above is an image of the water level in Bridgeport, Connecticut, over a 50 hour period in September. Using what you know about trigonometric functions, you will make sense of this information and do the job of an oceanographer for the day!
 - a. What initial observations do you have about the water levels shown by the graph?
 - b. Based on the image, what is the period for this graph? Explain how you know.
 - c. Use Desmos to create an equation to model the relationship between the water level in feet and the number of hours after midnight on September 1st. What equation did you come up with? Explain how you created and refined your equation to get a close fit to the graph shown in this problem.
 - d. How might swimmers, boaters, or fishers use the information in your graph to make informed decisions about their activities? Is there anybody else who might be interested in the information shown by the graph? If so, who?

Sample Learning Experience

Introduce the lesson by displaying the given image. Ask students: “What do you notice? What do you wonder?” Chart student responses close to the image, in a place that is visible to the whole class. Encourage them to elaborate in

order to make the ways they are attending to structure visible to themselves and their classmates. Invite them to record some of these responses and elaborate as they examine question 1a.

Give students time to engage in the remaining questions individually, followed by partner or small group work. Monitor and select varied approaches to the prompts to share for whole class discussion.

As students share their approaches with classmates, invite them to make sense of each other's work, particularly when engaging with how they approached Question 1c. Consider the following questions:

- What are some things you appreciate about this approach?
- What questions does this approach surface for you?
- What can you take from this explanation to apply to your own work?

Following the sharing and discussion, give students an opportunity to revise their work or build upon their work to share with each other again.

Task 2: Modeling Tides Student Work

Example 1:

Insert image of student work here

Example 2:

Insert image of student work here

Example 3:

Insert image of student work here

Criteria for Success

Conference and Provide Revision Support	Accept with Revision	Accept
The student's artifact shows an emerging understanding of the expectations of the indicator(s). After conferencing and additional instruction/learning, the student may provide a revised or different artifact as evidence of the indicator(s).	The student's artifact is approaching a full understanding of the expectations of the indicator(s). The artifact may contain execution errors that should be corrected in revision. The student may revise the selected artifact or submit a different artifact.	The student's artifact demonstrates evidence that they have met the expectations of the indicator(s).