

M203: Polynomial and Rational Expressions, Functions, and Equations Post-Badge Task

Reflecting on Badge Learning

Now that you have learned more about polynomial and rational expressions, functions, and equations, respond to the questions that follow. These are the same questions you responded to before the start of this badge.

The Task

QUESTION 1 Below are two graphs of polynomial functions.

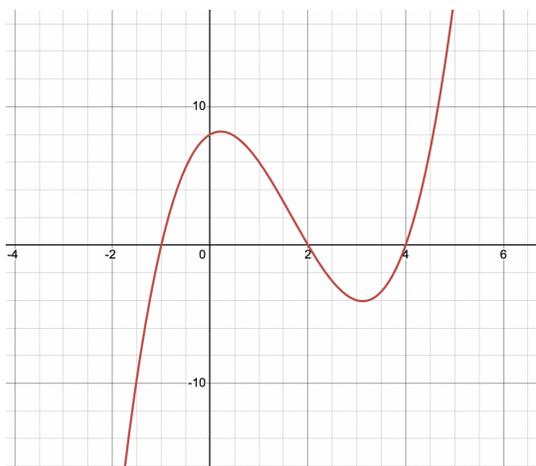
- Compare some of the key features of both graphs (including but not limited to their x-intercepts, end behavior, and general shape).
- Here are two polynomial functions, f and g , that correspond to the provided graphs. Identify which function corresponds to which graph and explain your reasoning.

$$f(x) = (x - 2)(x + 1)(x - 4)$$

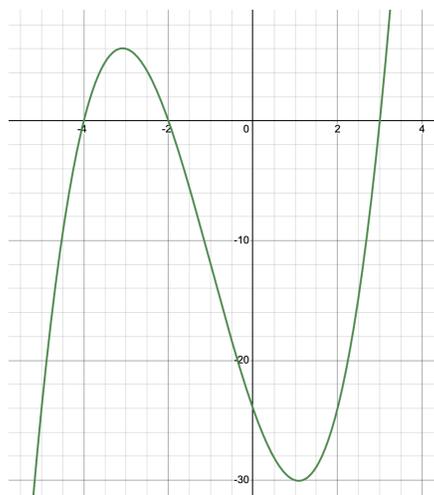
$$g(x) = x^3 + 3x^2 - 10x - 24$$



Graph 1:



Graph 2:



Key Feature Comparison:

Answers will vary. Possible student response:

The zeros for graph 1 include $x = -1, 2,$ and 4 . The zeros for graph 2 include $x = -4, -2,$ and 3 . The local max is about 8.2 and the local min is about -4 for graph 1; these are both larger than graph 2's local max and min at about 6 and about -30 . Both graphs have end behavior of negative, positive. Both graphs are continuous with two turning points.

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Graph 1 Function:

$$f(x) = (x - 2)(x + 1)(x - 4)$$

Justification:

The zeros being $x = -1, 2,$ and 4 are apparent in the equation. If we expand

$f(x)$, we have $f(x) = x^3 - 5x^2 + 2x + 8$ which confirms the y-intercept of 8 .

Graph 2 Function:

$$g(x) = x^3 + 3x^2 - 10x - 24$$

Justification:

The y-intercept of -24 is apparent in both the equation and the graph.

QUESTION 2 The function $h(x)$ represents another polynomial function. Its graph indicates that it has the following features:

- A degree of 4
- Zeros at $x = -3, x = 1, x = 4$
- A negative leading coefficient
- A y-intercept of 12

Which of the following equations could represent $h(x)$? Explain your reasoning.



A. $h(x) = -(x + 3)(x - 1)(x - 4)(x + 2)$

B. $h(x) = -(x + 3)(x - 1)^2(x - 4)$

C. $h(x) = (x + 3)(x - 1)^2(x - 4)$

D. $h(x) = -2(x + 3)(x - 1)(x - 4)$

Answers will vary. Possible student response:

D is not the correct choice because it does not have a degree of 4. A is not the correct choice because the y-intercept is -24 . C is not the correct choice because it does not have a negative leading coefficient.

B is correct because it has a degree of 4, zeros at $x = -3, 1,$ and 4 , a negative leading coefficient, and $h(0) = 12$ meaning the y-intercept is 12.

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QUESTION 3 Amber drew a graph of a polynomial that has exactly five zeros. Using this information, answer the following questions:

- Amber is exploring ways to write a 5th degree polynomial using those five zeros. Write a 5th degree polynomial equation that has exactly five zeros.
- Amber is exploring other ways to write a polynomial using those five zeros, this time she wants to have a degree that is not 5. Is this possible? If so, provide an example.



Answers will vary. Possible student response:

A polynomial function that has five zeros with a degree of 5 is

$$f(x) = x(x - 1)(x - 2)(x - 3)(x - 4)$$

The zeros are $x = 0, 1, 2, 3, 4$

Yes, it is possible. Here is a polynomial function that has five zeros with a degree of 7.

$$f(x) = x(x - 1)^3(x - 2)(x - 3)(x - 4)$$

The zeros are $x = 0, 1, 2, 3, 4$. The zero at $x=1$ has a multiplicity of 3.

QUESTION 4 Two competing businesses, Business K and Business J, tracked their yearly profits (in thousands of dollars) and found that the following models best fit their profits over the last year:

$$k(x) = x^3 + 5x^2 - 14x$$

$$j(x) = x^3 + 2x^2 - 3x - 36$$

- What do you **notice and wonder** about the two models? What information in the text above supports what you notice?
- How might you use these functions to compare the profits between Business K and Business J?
- Find the difference between k and j .
- The resulting polynomial is always positive; justify why this is. What does this tell you about the profits of Business K compared to Business J over the last year?



A. Answers will vary. Possible student response:

I notice both models are third degree functions, where the value of the function represents the profit each company made and x represents time.

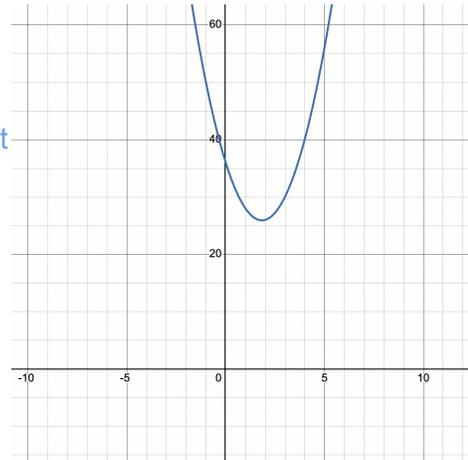
B. Answers will vary. Possible student response:

I looked at the two functions and noticed that we can ignore the x^3 part because they are the same. Then when you look at $5x^2$ and $2x^2$, you can see that $5x^2 > 2x^2$, so $k(x) > j(x)$.

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$$\begin{aligned} \text{C. } k(x) - j(x) &= x^3 + 5x^2 - 14x - (x^3 + 2x^2 - 3x - 36) \\ &= x^3 + 5x^2 - 14x - x^3 - 2x^2 + 3x + 36 \\ &= 3x^2 - 11x + 36 \end{aligned}$$

D. Answers will vary. Possible student response:
This polynomial will always be positive because if we graph it, we see that the absolute minimum is at about 26. This means all values of the function are above zero, so they are positive. This tells us that Business K made more money than Business J over the last year.



QUESTION 5 What else did you learn that was not reflected in Questions 1-4?



Answers will vary. Possible student responses:

- I can do other operations on polynomials, such as division.
- I can factor polynomials.
- I can graph polynomial functions.

QUESTION 6 Compare your answers to these questions now to your answers from the earlier attempt. What do you notice about your progress?



Answers will vary. Look for students to explain what they know now compared to what they knew when responding to the pre-test.