

Annotated Examples: M204 Exponential and Logarithmic Functions and Equations

The examples that follow are intended to illustrate how the learning principles are used to support students' engagement with the content and practices outlined in this badge. These examples do not provide comprehensive coverage of those expectations, but rather elevate some of the learning principles that are less likely to be part of published curricular materials for mathematics instruction. The examples that follow were developed by the Math Badging writing team, unless otherwise specified. These are a small sample of types of learning activities that can be done with students, both in and out of a traditional classroom setting.

Example 1: Asked and Answered

Students are given a prompt like this:

Consider the following scenarios.

1. There are currently 289 million registered cars in the United States, increasing by about 1.5% annually.
2. For the original strand of COVID-19, one infected person was responsible for further infecting about two people during their infectious period. The spread of the virus continued in that pattern. The current US population is about 330 million people.
3. There are 20,000 fish in an infected pond. The population of fish declines to 8,000 in four hours.

For each of the scenarios above, write down questions you could ask about the situation. Write as many questions as you can think of.

As a group, you will choose some interesting questions to answer. If you would need to do calculations, make a sketch, or draw a graph in order to answer a question, then it's probably an interesting question.

Write a report to explain your questions and answers. The report should include:

- a context for your situation. Write a story that includes all the relevant information. If the description of the situation is missing some details, fill them in however you want.
- an equation to describe the situation, with an explanation of what each part of the equation represents.
- a labeled graph of the situation, including the questions you chose and your answers. Include your process for finding the answers. Say what the answers mean in the context of this situation.

Adapted from [Illustrative Mathematics Modeling Prompt](#) (Illustrative Mathematics, 2019)

Sample Learning Activity

Provide time for students to consider the question posed and independently develop a response before they are encouraged to work with a partner or small group.

Monitor and select students to share their approach as they work on these prompts. Consider using the following questions to facilitate the discussion:

- *How would you describe the relationships captured in your preliminary work: linear, exponential, or something else? How can you tell?*
- *What's promising about this approach?*
- *What questions does this approach surface for you?*
- *What can you take from this approach to apply to your own work?*

Provide time for students to revise and refine their approach. Give students additional time to explore their ideas and conduct additional research, as well as to prepare their report. Reports can take any form that students feel is appropriate: traditional presentation, TED Talk style, a video, an animation, or any other form that makes sense to them.

As students showcase their work, invite classmates to engage with the presentation. Consider asking the following questions:

- *What is something you appreciate about the presentation?*
- *What about this reasoning makes sense?*
- *Is there anything you disagree with?*
- *What questions surface for you?*

Provide time for students to revise their work.

In this example, students are:

- given a cognitively demanding task. Students are asked to generate questions of interest they want to explore. As they generate questions, they have the opportunity to explore possible relationships and representations, which sets the stage for the research component (LP 1).
- using the varied structures that give opportunities to determine an approach by working independently, sharing, and collaborating with a partner or classmates. Students also have opportunities to refine and revise their response based on those interactions (LP 2).
- making sense of another person's line of mathematical reasoning as well as critiquing the mathematical reasoning that surfaces. As students consider the type of relationships they're formulating, they deepen their understanding of exponential relationships (LP 3).
- given the opportunity to develop their mathematical identity since their ideas and reasoning are the foundation of the discussion (LP 4).
- developing a sense of identity and agency as they work to make sense of self-selected investigations, gaining insight into the work various career paths do when using mathematics to model relationships (LP 5 and LP 6).

- leveraging technology to explore and determine exponential equations or functions to best answer their questions (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- Creative Process:** [Asking questions](#) (OT.Creat.1.a)

Example 2: Tuition and Income

Students are given a prompt like this:

The tuition at a college was \$30,000 in 2012, \$31,200 in 2013, and \$32,448 in 2014. The tuition has been increasing by the same percentage since the year 2000.

- The equation $c(t) = 30,000 \cdot (1.04)^t$ represents the cost of tuition, in dollars, as a function of t , the number of years since 2012. Explain what the 30,000 and 1.04 tell us about this situation.*
- What is the percent increase in tuition from year to year?*
- What does $c(3)$ mean in this situation? Find its value and show your reasoning.*
- Write an expression to represent the cost of tuition in 2007. How much did tuition cost that year?*
- Typically, it takes students about five years to complete a bachelor's degree.*
 - If you were entering this college, what is the anticipated tuition for your first year?*
 - How much would you anticipate the total cost in tuition would be to complete your bachelor's degree?*
 - Select a career path that requires a bachelor's degree. Research how much someone in that career makes over time. Create an equation or graph of the potential earnings.*
 - Based on this work, how would you describe the impact of earning a bachelor's degree in that career path given the cost of tuition?*
 - Examine some other careers that don't require a bachelor's degree. Can you find any with similar earning potential?*
- Based on your work, what do you think about the cost of a college education, considering both the investment involved and expected financial outcomes after graduation? What are other factors you might consider?*

Your task is to develop a public service announcement (PSA) in the form of an infographic or advertisement (suitable for radio, social media, or television) to inform a target audience about the cost of college and earning wages.

Your PSA should include information that informs your audience about:

- *costs of college tuition*
- *earning potential of various career options*
- *your claim*

Your PSA should:

- *identify relevant quantities*
- *use appropriate graphical displays of these quantities*
- *include any other information that helps justify your claim*

Adapted from Illustrative Mathematics Alg 2 Unit 4 Lesson 2 (Illustrative Mathematics, 2019)

Sample Learning Activity

After some individual work time where students develop an approach to the prompt, encourage them to share and compare approaches. Monitor and select a couple of approaches to highlight and reflect on progress made at this point. Look for student work that indicates students are beginning to attend to the structure of the equation. Make technological tools available so that students can reason using tables and graphs as they make sense of the structure of the exponential function given.

Questions that can be posed to student pairs as they work:

- *What do the numbers in the equation represent?*
- *What is the growth rate in the equation?*
- *How would you describe the change in tuition costs?*

Give selected pairs of students time to share responses and invite classmates to reflect on the following questions:

- *What's promising about this approach?*
- *What questions does this approach surface for you?*
- *What can you take from this approach to apply to your own work?*

Provide time for students to revise their work. While they work, select additional pairs to share how they used the structure and relationship between the equations to determine which sets of equations have the same answer. Give students additional time to revise their work. Repeat this process as students engage with Question 5.

Give students time to conduct further research to answer some of their questions. As students develop their PSA, provide opportunities for students to share progress with classmates. Consider the following questions to facilitate discussion:

- *What's promising about the planned PSA?*
- *What questions surface for you?*
- *Do you think your data supports your claim? Why or why not?*
- *What can you take from this approach to apply to your own work?*

Give students time to conduct further research and revise their PSA. As students finalize their PSA, share products with classmates and at your site. Explore ways to partner with local businesses and radio and television stations in order to broadcast student work. Leverage social media outlets and electronic applications that your site has available.

In this example, students are:

- given a cognitively demanding task. They are asked to analyze components of an exponential function. Students have multiple entry points to reason about these equations in various ways (LP 1).
- working collaboratively in pairs to complete the task, allowing for social interaction with peers. Students are also encouraged to ask questions to their peers to understand each other's thinking process and revise their own thinking (LP 2).
- attending to the structure of exponential functions and working to make sense of the relationships between the starting value, growth rate, and exponent. In so doing, students learn to recognize that different forms are helpful for different purposes (LP 3).
- encouraged to develop skills of expert learners, thereby developing agency, as they reflect on their approach to the prompt and incorporate new ways of thinking about this from their peers (LP 4).
- developing a sense of identity and agency as they work to make sense of costs of college and potential earnings to gain insights into the cost-benefits of a college education. In doing this work, students broaden their ideas about what mathematics is, how it is used, and who it is for (LP 5 and LP 6).
- leveraging technology to examine the nature of exponential functions (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Community Advocacy:** [Case-making](#) (GC.SAg.1.c)

Example 3: Viral Load

Students are given a prompt like this:

When a virus invades our body, it gets our body to make so many copies of it that it disrupts normal functioning of our body at the cellular level and makes us sick. Watch this video from [Life Noggin- Just How Fast Does a Virus Spread?](#) to learn more. As the virus spreads to other cells in our body, the viral load in our body increases. The viral load measures how much viral RNA is present in some body fluid, such as blood or saliva. The unit of measure is in copies per mL using \log_{10} .

Part 1: Making Sense of Log Base 10

Let's explore how this works. Study the table below.

x	10	100	1,000
$\log_{10} x$	1	2	3

1. What do you notice? How are the numbers changing?
2. How would you describe what $\log_{10} x$ is?
3. What would the graph of $y = \log_{10} x$ look like?
4. What do you anticipate this has to do with the concept “viral load”?

Part 2: Making Connections to the Medical Field

A group of researchers set out to understand how viral load is associated with the risk of COVID-19 transmission. They found that the median viral load among the cases they looked at was “ $5.6\log_{10}$ copies of RNA per mL of saliva.” [source: [SARS-CoV-2 viral load is associated with risk of transmission to household and community contacts | BMC Infectious Diseases.](#)]

5. What about this expression makes sense to you at this point?
6. What questions do you have?

The medical field uses \log_{10} both as a label and in its mathematical sense. Examine the table below to see how.

$W\log_{10}$	$1\log_{10}$	$2\log_{10}$	$3\log_{10}$	$4\log_{10}$	$5.6\log_{10}$
number of copies per mL of saliva	10	100	1000		

7. Complete the table using patterns that you see.
8. The same researchers found that a small portion of cases had viral loads greater than $8\log_{10}$. How many copies per mL is that? How much greater is this viral load than $4\log_{10}$?

Part 3: Reflecting on Your Learning

9. What is something you’ve learned about the meaning of logarithms?
10. What do you think the significance of these numbers is? Why would anyone be interested in understanding the concept of viral load?
11. What is something you’ve learned about yourself, as you made sense of these ideas?

Sample Learning Activity

Launch this experience by asking students what’s the largest number they can imagine. You can use numbers they share to lead into the idea of how viruses make us sick, possibly asking: *do you think your body can hold that many copies of a virus?* Take a poll and make note of some of the ideas that surface in a visible space, like a poster or whiteboard. Reference these ideas as applicable throughout this journey. Listen for ideas that the Life Noggin video addresses as well as ideas that can transition to the concept of viral load.

Show the video and amplify the segment where the virus gets our cells to make multiple copies. Share with students that it is this phenomenon (replication of RNA) that experts in the medical field work to quantify with a unit of measure called *viral load*. Encourage students to share questions they have and, as time permits, allow students to research this topic.

Consider using these questions to debrief this segment:

- *What is something that you learned that was new?*
- *What surprised you?*

Part 1: Making Sense of Log Base 10

Frame this learning activity by sharing that we will be working to understand how viral load is measured, and this entails very large numbers. Give students time to work on Part 1 individually, prior to inviting them to work with a partner or small groups.

As students work, consider posing these questions:

- *What do the values in the log function represent?*
- *How is the function growing? Where do we see that in the table? Graph?*
- *What connections are there between the different representations?*

Monitor and select varied approaches and responses to share out to the class. Consider using the following questions to facilitate discussion:

- *What about this makes sense to you?*
- *What questions does this work surface for you?*
- *What can you take from this approach to apply to your own work?*

Provide time for students to revise and refine their work.

Part 2: Making Connections to the Medical Field

Give students quiet work time before inviting students to partner or work with a small group. Consider pausing after Questions 4 and 5 to give space for students to share their current understandings and work to develop a shared understanding of the context. Capture ideas students share on a visible surface such as chart paper or a whiteboard.

Encourage students to engage in the next set of questions as a way to address some of the questions they've surfaced as well as to confirm some of the ideas they have.

As students work, pose the following questions:

- *What similarities do you see between the work you did in Part 1 and what you're looking at here?*
- *How can you use what you know from Part 1 to explain what you see happening in this table?*
- *What tools might be helpful to you: equation? table? graph?*
- *What connections are there between the different representations?*

Monitor and select students to share their thinking with the class. Use suggested questions as you see fit. Look for opportunities to connect to students' initial ideas and questions captured in early parts of this experience.

Part 3: Reflecting on Your Learning

Give students an opportunity to reflect on their learning and to share with others. As needed, provide time for students to revise and refine their work.

In this example, students are:

- given a cognitively demanding task. They have opportunities to use what they understand about exponentiation and multiplicative growth patterns to make sense of logarithms (LP 1).
- working collaboratively in pairs and sharing their findings with the class. Students are encouraged to ask questions to their peers to understand one another's thinking process (LP 2).
- engaging in extending their understanding of exponentiation and multiplicative growth patterns to develop an understanding of logarithms (LP 3).
- given the opportunity to develop their mathematical identity since their ideas and reasoning are the foundation of the discussion (LP 4).
- given opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for as they examine the mathematical underpinnings of logarithms and their use in the medical field (LP 5).
- engaging with an authentic use of logarithms in the context of viral loads and examining why this matters (LP 6).
- leveraging technology to explore logarithmic expressions and functions as they develop conceptual understanding of the logarithmic functions (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Scientific Investigation:** [Modeling systems](#) (FL.MST.4.c)

References

Bhavnani, D., James, E.R., Johnson, K.E. *et al.* SARS-CoV-2 viral load is associated with risk of transmission to household and community contacts. *BMC Infect Dis* 22, 672 (2022).
<https://doi.org/10.1186/s12879-022-07663-1>

Illustrative Mathematics. “Exponential Situations.” *Modeling Prompt*, Illustrative Mathematics, 2019,
https://im.kendallhunt.com/HS/teachers/3/modeling_prompts/5/preparation.html.

Illustrative Mathematics. “Representations of Growth and Decay.” *Illustrative Mathematics, Algebra 2, Unit 4, Lesson 2*, Illustrative Mathematics, 2019,
<https://im.kendallhunt.com/HS/teachers/3/4/2/index.html>.

Life Noggin. “Just How Fast Does A Virus Spread?” *YouTube*, Life Noggin, 27 July 2017,
https://www.youtube.com/watch?v=L_WaiVi3Aik.