

# Annotated Examples: M104 Modeling with Functions of Exponential Type

The examples that follow are intended to illustrate how the learning principles are used to support students' engagement with the content and practices outlined in this badge. These examples do not provide comprehensive coverage of those expectations, but rather elevate some of the learning principles that are less likely to be part of published curricular materials for mathematics instruction. The examples that follow were developed by the Math Badging writing team, unless otherwise specified. These are a small sample of types of learning activities that can be done with students, both in and out of a traditional classroom setting.

## Example 1: Movie Box Office

Students are given a problem like this:

*The original Superman movie came out in 1978 and made about \$300 million at the box office. If the yearly inflation rate is about 4%, how much money would this be equivalent to in 2022?*

### Sample Learning Activity

Present the fact that the 1978 Superman movie made \$300 million at the box office. Ask students to hypothesize how much this might be in today's money. As a way to further engage interest, allow time for students to research a couple of box office amounts for other superhero movies.

Students are given time to consider the question and develop an answer independently, before having an opportunity to share their response with others. Eventually, responses—some of which could be partially complete—that highlight various solution pathways are selected for sharing with the whole class. As you monitor students, look for approaches where students:

- use various representations such as a table, expression, equation, graph, etc.
- use different models (linear, exponential, other).

One sample table is illustrated below:

Year	Revenue (millions of dollars)
1978	300
1979	312
1980	324.48

One sample expression is illustrated below:

$$300 \cdot 1.04^t$$

If graphing as a model representation did not arise, encourage students to graph their model by hand or using technology. Ask students to compare their graphs to each other's.

Instead of quickly asking students to identify an answer or approach, give students time to share responses to questions such as:

- *What's promising about this approach?*
- *What questions does this approach surface for you?*
- *Do you think the model is reasonable? Why or why not?*
- *What can you take from this approach to apply to your own work?*
- *What additional questions could be answered about this context using some of the models we've seen? Are some models more helpful than others in answering these additional questions?*

Following the discussion, students are given an opportunity to reflect on and revise their work. Finally, one or more reasonable models are revealed explicitly, and students have an opportunity to reflect on the reasonable models and revise their work.

In this example, students are:

- given a cognitively demanding task. It requires that students model exponential growth as they determine the equivalent movie revenue in "today's dollars," but does not specify a particular method. To increase the cognitive complexity of the task, it is possible to incorporate more aspects of the modeling cycle. For example, there might be more emphasis on "interpreting the results" by asking students to consider a particular audience that might be interested in the results of such a problem. All learners are supported as the task supports peer learning: students have an opportunity to share and discuss each other's responses, followed by time to reflect and revise their own work (LP 1).
- set up to have social interactions, as they share and discuss their work with each other. These interactions may happen live in a physical classroom or asynchronously in a remote setting (LP 2).
- encouraged to have agency, as they choose, reflect on, and refine their approaches. We can further increase students' agency in learning by offering a more open task (LP 4). For example, tasks can be less defined and require more assumptions on the part of the modeler: *A 30-year-old movie made about \$300 million at the box office when it came out. How would you compare this amount of money to the revenue from a movie that came out today?*

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Interpreting Information:** [Information management](#) (OT.Crit.1.c)

### Example 2: Tracking COVID Cases

Students are given a table that shows the number of positive COVID-19 cases and the weekly rate of increase of infections for different US ZIP codes in April 2020:

ZIP Code	Number of Infections	Rate of Increase (4/2/20 - 4/9/20)
10025	343	5.6%
10029	462	26.4%
10035	56	3.2%

### Sample Learning Activity

Students are asked to work in triads to use a Google Sheet (or other spreadsheet software) to model what the number of infections will be on July 1, 2020.

As students work, they respond to questions such as:

- *What formulas are you using in your Google Sheet? Write them as Google Sheet formulas and algebraic equations.*
- *What assumptions are you making as you develop these formulas?*
- *Use online search tools to locate census data to learn more about these ZIP codes. What do you notice? What do you wonder?*
- *Look up the actual rate of infection in each ZIP code on July 1, 2020. How accurate was your model? How would you change it? What additional assumptions or parameters might have caused the model to change?*
- *What does the work you have done suggest about the need to have people who are constantly studying data and refining models as part of their work?*
- *What additional questions surface for you as a result of the work above?*
- *How could the work you have done be used to inform policymakers, such as health or government officials?*

Following the discussion, students are given an opportunity to reflect on and revise their work. Finally, one or more reasonable models are revealed explicitly, and students have an opportunity to reflect on the reasonable models and revise their work.

In this example, students are:

- given a cognitively demanding task. It requires that students model exponential growth as they estimate the number of COVID-19 infections in different ZIP codes. Students are also asked to consider improvements to their model. They further engage in thinking about why models change as the parameters and assumptions change (LP 1).
- set up to have social interactions, as they work in triads to model the data (LP 2).

- able to explore areas of interests and develop a lens for aspirations, as they consider the work of an epidemiologist (LP 4).
- applying mathematics in a relevant, real-world situation. Students will notice how factors like population density and demographics correlate with infection rates, leading to considerations of policies that could mitigate the disproportionate impact of the pandemic on the population (LP 6).
- able to use technology as a tool, as they employ Google Sheets to perform their analysis (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Productive Collaboration:** [Navigating Roles](#) (GC.IS.4.b)

## References

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