

# M203: Polynomial and Rational Expressions, Functions, and Equations

## Illustrative Mathematics—Algebra 2 Unit 2- Polynomial and Rational Functions

### Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4)**. This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.

### Learning Principles

**Engage with cognitively demanding tasks in heterogeneous settings (LP 1).** Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

**Engage in social activities (LP 2).** Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one’s thinking has excellent value in solidifying one’s knowledge.

**Build conceptual understanding through reasoning (LP 3).** Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

**Have agency in their learning (LP 4).** Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students’ identities and interests to the fore and build on the strengths that they bring to the learning space.

**View mathematics as a human endeavor across centuries (LP 5).** Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

**See mathematics as relevant (LP 6).** Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

**Employ technology as a tool for problem-solving and understanding (LP 7).** Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense

of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

## Content and Practice Expectations

| Content and Practice Expectations  | Indicators<br>Choose an artifact where you...   |
|--|---|
| 203.a: Interpret the structure of polynomial and rational expressions.                                 | i. interpret how parts of an expression, such as terms, factors, and coefficients relate to a real-world situation.                         |
|  | ii. interpret complicated expressions by viewing one or more of their parts as a single entity.   |
|  | iii. use the structure of an expression to identify ways to rewrite it.   |
| 203.b: Reason about and perform operations on polynomials.   | i. demonstrate understanding that polynomials are closed under the operations addition, subtraction, and multiplication.                    |
|  | ii. add, subtract, and multiply polynomials.  |
| 203.c: Understand the relationship between the zeros and factors of polynomials.                       | i. identify zeros of polynomials when suitable factorizations are available.  |
|  | ii. use the zeros to construct a rough graph of a function defined by a polynomial.   |
| 203.d: Use polynomial identities to solve problems.  | i. prove polynomial identities and use them to describe a numerical relationship.   |
| 203.e: Rewrite rational expressions.   | i. rewrite simple rational expressions in different forms.  |
| 203.f: Interpret polynomial and rational functions that arise in applications in terms of the context. | i. interpret key features of graphs and tables in terms of the quantities for a function that models a relationship between two quantities. |

| Content and Practice Expectations | Indicators<br>Choose an artifact where you...   |
|-----------------------------------|---|
|                                   | ii. sketch graphs showing key features given a verbal description of the relationship between quantities.               |
|                                   | iii. calculate and interpret the average rate of change of a polynomial or rational function over a specified interval. |
|                                   | iv. relate the domain of a function to its graph, and where applicable, to the quantitative relationship it describes.  |

## Unit Overview

|           | <a href="#">L1</a> | <a href="#">L2</a> | <a href="#">L3</a> | <a href="#">L4</a> | <a href="#">L5</a> | <a href="#">L6</a> | <a href="#">L7</a> | <a href="#">L8</a> | <a href="#">L9</a> | <a href="#">L10</a> | <a href="#">L11</a> | <a href="#">L12</a> | <a href="#">L13</a> | <a href="#">L14</a> | <a href="#">L15</a> | <a href="#">L16</a> | <a href="#">L17</a> | <a href="#">L18</a> | <a href="#">L19</a> | <a href="#">L20</a> | <a href="#">L21</a> | <a href="#">L22</a> | <a href="#">L23</a> | <a href="#">L24</a> | <a href="#">L25</a> | <a href="#">L26</a> | <a href="#">M</a><br><a href="#">I</a> |   |
|-----------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|--|---|
| 203.a.i   | X                  | X                  |                    |                    |                    | X                  |                    |                    |                    |                     |                     |                     |                     |                     |                     | X                   | X                   | X                   | X                   | X                   | X                   |                     |                     |                     |                     | X                   | X                                      | X |
| 203.a.ii  | X                  | X                  | X                  |                    |                    | X                  |                    | X                  | X                  | X                   |                     |                     | X                   |                     |                     | X                   | X                   | X                   | X                   | X                   | X                   | X                   |                     |                     |                     | X                   | X                                      | X |
| 203.a.iii | X                  | X                  | X                  |                    |                    | X                  |                    |                    | X                  |                     |                     | X                   | X                   | X                   |                     | X                   |                     | X                   | X                   | X                   | X                   | X                   | X                   | X                   | X                   | X                   | X                                      |   |
| 203.b.i   |                    |                    |                    | X                  |                    | X                  |                    |                    |                    |                     |                     | X                   | X                   | X                   |                     |                     |                     |                     |                     |                     |                     |                     |                     | X                   |                     | X                   |  |   |
| 203.b.ii  |                    |                    |                    | X                  |                    | X                  |                    |                    |                    |                     |                     | X                   | X                   | X                   |                     |                     |                     |                     |                     |                     |                     |                     |                     | X                   |                     | X                   |  |   |
| 203.c.i   | X                  |                    | X                  |                    | X                  | X                  | X                  |                    | X                  | X                   |                     | X                   |                     | X                   |                     |                     |                     |                     |                     |                     |                     |                     |                     |                     |                     |                     |  |   |
| 203.c.ii  |                    |                    | X                  |                    | X                  | X                  | X                  |                    | X                  | X                   |                     | X                   |                     | X                   |                     |                     |                     |                     |                     |                     |                     |                     |                     |                     |                     |                     |  | X |
| 203.d.i   |                    |                    |                    |                    |                    |                    |                    |                    |                    |                     |                     |                     |                     |                     |                     |                     |                     |                     |                     |                     |                     |                     |                     | X                   | X                   | X                   | X                                      |   |
| 203.e.i   |                    |                    |                    |                    |                    |                    |                    |                    |                    |                     |                     |                     |                     |                     |                     | X                   |                     | X                   | X                   | X                   | X                   | X                   |                     | X                   |                     | X                   |  |   |

|           |   |   |   |  |  |   |  |   |   |   |  |  |  |   |   |   |   |   |   |   |   |  |  |  |   |  |   |
|-----------|---|---|---|--|--|---|--|---|---|---|--|--|--|---|---|---|---|---|---|---|---|--|--|--|---|--|---|
| 203.f.i   | X | X | X |  |  | X |  | X |   | X |  |  |  | X |   | X | X | X | X | X | X |  |  |  | X |  | X |
| 203.f.ii  | X | X |   |  |  |   |  |   | X |   |  |  |  | X |   | X | X | X |   | X |   |  |  |  |   |  | X |
| 203.f.iii |   | X |   |  |  |   |  |   |   |   |  |  |  |   |   |   |   |   |   |   |   |  |  |  |   |  | X |
| 203.f.iv  | X | X |   |  |  | X |  |   |   |   |  |  |  |   | X |   | X |   |   |   |   |  |  |  |   |  | X |

Note: Columns in gray indicate a recommendation to delete the lesson

## Guidance for Lesson Priority

| Unit.Lesson | Priority<br>D - Delete<br>M - Modify<br>K - Keep  | Content and Practice Expectations (CPE)  | Learning Principles (LP) | Additional Lesson Information  |
|-------------|---|--|--------------------------|--|
| 2.1         | K<br>Attend to 203.a.iii, look for a variety of equivalent expressions in Activity 1.3. | 203.a.i<br>203.a.ii<br>203.a.iii<br>203.c.i<br>203.f.i<br>203.f.ii<br>203.f.iv   | LP1, LP2, LP3, LP6, LP7  | <b>Context:</b> Building a box<br><br><b>Math focus:</b> Finding the volume; creating a polynomial                             |
| 2.2         | K   | 203.a.i<br>203.a.ii<br>203.a.iii<br>203.f.i<br>203.f.ii<br>203.f.iii<br>203.f.iv | LP1, LP3, LP6, LP7       | <b>Context:</b> Investing money<br><br><b>Math focus:</b> Using a polynomial to model a situation                              |
| 2.3         | K   | 203.a.ii<br>203.a.iii<br>203.c.i<br>203.c.ii<br>203.f.i                          | LP1, LP2, LP3, LP7       | <b>Context:</b> Purely mathematical<br><br><b>Math focus:</b> Identifying characteristics of polynomial graphs and expressions |
| 2.4         | K   | 203.b.i<br>203.b.ii  | LP2, LP3                 | <b>Context:</b> Purely mathematical<br><br><b>Math focus:</b> Adding, subtracting,   |

| Unit.Lesson | Priority<br>D - Delete<br>M - Modify<br>K - Keep   | Content and Practice Expectations (CPE)   | Learning Principles (LP) | Additional Lesson Information   |
|-------------|--|---|--------------------------|---|
|             |  |   |                          | multiplying polynomials   |
| 2.5         | K  | 203.c.i<br>203.c.ii   | LP1, LP2, LP3            | <b>Context:</b> Purely mathematical<br><br><b>Math focus:</b> Finding the zeros from the factored form                                      |
| 2.6         | K<br>Consider asking students to describe the domain of the function in focus on Activity 6.2 to get to full extent of 203.f.iv. | 203.a.i<br>203.a.ii<br>203.a.iii<br>203.b.i<br>203.b.ii<br>203.c.i<br>203.c.ii<br>203.f.i<br>203.f.iv | LP1, LP2, LP3, LP6, LP7  | <b>Context:</b> Revisits box from Lesson 1<br><br><b>Math focus:</b> Identifying features of a polynomial using standard and factored forms |
| 2.7         | K  | 203.c.i<br>203.c.ii   | LP1, LP3, LP7            | <b>Context:</b> Choosing proper window settings<br><br><b>Math focus:</b> Writing polynomials given the zeros                               |
| 2.8         | K  | 203.a.ii<br>203.f.i   | LP1, LP3, LP7            | <b>Context:</b> Purely mathematical<br><br><b>Math focus:</b> Understanding why the leading term determines end behavior                    |

| Unit.Lesson | Priority<br>D - Delete<br>M - Modify<br>K - Keep   | Content and Practice Expectations (CPE)                  | Learning Principles (LP) | Additional Lesson Information  |
|-------------|--|--|--------------------------|--|
| 2.9         | K  | 203.a.ii<br>203.a.iii<br>203.c.i<br>203.c.ii<br>203.f.ii | LP1, LP2, LP3, LP7       | <b>Context:</b> Purely mathematical<br><br><b>Math focus:</b> Using the leading term to determine end behavior                 |
| 2.10        | K  | 203.a.ii<br>203.c.i<br>203.c.ii<br>203.f.i               | LP1, LP2, LP3, LP7       | <b>Context:</b> Purely mathematical<br><br><b>Math focus:</b> Using zeros and multiplicities to sketch a graph of a polynomial |
| 2.11        | D<br>Not aligned to expectations of this badge and not needed for coherence.                         |  |                          | <b>Context:</b> Purely mathematical<br><br><b>Math focus:</b> Finding where two polynomials intersect                          |
| 2.12        | K<br>Division of polynomials is the context for engaging in 203.b. Amplify the underlying reasoning. | 203.a.iii<br>203.b.i<br>203.b.ii<br>203.c.i<br>203.c.ii  | LP1, LP2, LP3            | <b>Context:</b> Purely mathematical<br><br><b>Math focus:</b> Dividing polynomials using diagrams                              |

| Unit.Lesson | Priority<br>D - Delete<br>M - Modify<br>K - Keep   | Content and Practice Expectations (CPE)  | Learning Principles (LP) | Additional Lesson Information  |
|-------------|--|--|--------------------------|--|
| 2.13        | K<br>Division of polynomials is the context for engaging in 203.b. Amplify the underlying reasoning. | 203.a.iii<br>203.b.i<br>203.b.ii   | LP1, LP2, LP3            | <b>Context:</b> Purely mathematical<br><br><b>Math focus:</b> Dividing polynomials using long division                             |
| 2.14        | K  | 203.a.ii<br>203.a.iii<br>203.b.i<br>203.b.ii<br>203.c.i<br>203.c.ii<br>203.f.i<br>203.f.ii | LP1, LP2, LP3            | <b>Context:</b> Purely mathematical<br><br><b>Math focus:</b> Rewriting a polynomial in factored form starting from a known factor |
| 2.15        | D<br>Not aligned to expectations of this badge and not needed for coherence.                         |  |                          | <b>Context:</b> Purely mathematical<br><br><b>Math focus:</b> Remainder theorem  |
| 2.16        | K  | 203.a.i<br>203.a.ii<br>203.a.iii<br>203.e.i<br>203.f.i                                     | LP1, LP2, LP3, LP7       | <b>Context:</b> Minimizing surface area of a cylinder<br><br><b>Math focus:</b> Writing rational functions                         |

| Unit.Lesson | Priority<br>D - Delete<br>M - Modify<br>K - Keep | Content and Practice Expectations (CPE)  | Learning Principles (LP) | Additional Lesson Information   |
|-------------|--|--|--------------------------|---|
|             |  | 203.f.iv   |                          |   |
| 2.17        | K  | 203.a.i<br>203.a.ii<br>203.f.i<br>203.f.ii                                     | LP1, LP2, LP3, LP6, LP7  | <b>Context:</b> Distance, rate, and time relationship for a bike ride<br><br><b>Math focus:</b> Identifying vertical asymptotes                         |
| 2.18        | K  | 203.a.i<br>203.a.ii<br>203.a.iii<br>203.e.i<br>203.f.i<br>203.f.ii<br>203.f.iv | LP1, LP2, LP3, LP6       | <b>Context:</b> Cost of publishing a book<br><br><b>Math focus:</b> Identifying horizontal asymptotes   |
| 2.19        | K  | 203.a.i<br>203.a.ii<br>203.a.iii<br>203.e.i<br>203.f.i<br>203.f.ii             | LP1, LP2, LP3, LP6       | <b>Context:</b> Combined fuel efficiency<br><br><b>Math focus:</b> Finding end behavior of rational functions by rewriting as $f(x) = q(x) + r(x)/b(x)$ |
| 2.20        | K  | 203.a.i<br>203.a.ii<br>203.a.iii<br>203.e.i                                    | LP1, LP2, LP3, LP6, LP7  | <b>Context:</b> Price of t-shirts; batting averages<br><br><b>Math focus:</b> Writing rational expressions  |

| Unit.Lesson | Priority<br>D - Delete<br>M - Modify<br>K - Keep | Content and Practice Expectations (CPE)                            | Learning Principles (LP) | Additional Lesson Information   |
|-------------|--|--|--------------------------|---|
|             |  | 203.f.i  |                          | that represent averages   |
| 2.21        | K  | 203.a.i<br>203.a.ii<br>203.a.iii<br>203.e.i<br>203.f.i<br>203.f.ii | LP1, LP2, LP3, LP6, LP7  | <b>Context:</b> Boat ride on river; parallel circuits<br><br><b>Math focus:</b> Writing and solving equations with simple rational expressions on both sides          |
| 2.22        | K  | 203.a.ii<br>203.a.iii<br>203.e.i                                   | LP1, LP2, LP3            | <b>Context:</b> Purely mathematical<br><br><b>Math focus:</b> Checking for extraneous solutions to rational equations   |
| 2.23        | K  | 203.a.iii<br>203.b.i<br>203.b.ii<br>203.d.i                        | LP1, LP2, LP3, LP5       | <b>Context:</b> Purely mathematical<br><br><b>Math focus:</b> Understanding what an identity is in mathematics; investigating the difference of squares and $x^n-1$ . |
| 2.24        | K  | 203.a.iii<br>203.d.i<br>203.e.i                                    | LP1, LP2, LP3, LP5       | <b>Context:</b> Sides of a right triangle; Egyptian fractions<br><br><b>Math focus:</b> Justifying why identities are true  |

| Unit.Lesson                                    | Priority<br>D - Delete<br>M - Modify<br>K - Keep   | Content and Practice Expectations (CPE)   | Learning Principles (LP)     | Additional Lesson Information  |
|--|--|---|------------------------------|--|
| 2.25   | K  | 203.a.i<br>203.a.ii<br>203.a.iii<br>203.b.i<br>203.b.ii<br>203.d.i<br>203.f.i   | LP1, LP2, LP3, LP5, LP6, LP7 | <b>Context:</b> Koch snowflake; antibiotics<br><br><b>Math focus:</b> Understanding why the geometric sum formula is true                                  |
| 2.26   | K  | 203.a.i<br>203.a.ii<br>203.a.iii<br>203.d.i<br>203.e.i                          | LP1, LP2, LP3, LP5, LP6, LP7 | <b>Context:</b> House sales; savings account<br><br><b>Math focus:</b> Using the geometric sum formula to solve problems                                   |
| Modeling Task- <u>The Paths of the Planets</u> | Two versions are given—the first three questions are the same, but the fourth gives data in one version and has students research to find the data in the second version. To increase alignment to CPEs listed, ask students to interpret key features of the graph and describe the domain. | 203.a.i<br>203.a.ii<br>203.c.ii<br>203.f.i<br>203.f.ii<br>203.f.iii<br>203.f.iv | LP1, LP2, LP3, LP5, LP6, LP7 | <b>Context:</b> Kepler’s relationship between planets’ orbital periods and their distance from the sun<br><br><b>Math focus:</b> Modeling with polynomials |

