

M103: Modeling with Functions of Quadratic Type

[Illustrative Mathematics–Algebra 1 Unit 6: Introduction to Quadratic Functions](#)

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4)**. This can be accomplished by learning about students’ interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum’s mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- The modeling cycle is not well represented, and there are few opportunities for students to identify a problem, formulate their own questions, or report. The use of [Math Language Routine 5: Co-Craft Questions](#) may be useful in building these opportunities into some of the existing modeling tasks within the unit. This is necessary to fully meet [Content and Practice Expectation 103.a](#).
- There are minimal opportunities to describe the domain of the function ([Indicator 103.b.iii](#)) built into the lessons. Consider explicitly asking students to reflect on the given context or situation and to identify a reasonable domain. Early in the unit, consider building conceptual understanding of domain before naming what students are identifying as finding the domain. For example, you may ask, “*What are some reasonable values for the input? What are reasonable values for the output?*” or “*Would the values (x, y) be a reasonable choice in this situation? Why or why not?*”, where the values for x and y do satisfy the equation, but extend beyond the domain. Spaces where we see an opportunity for this are indicated by a star ★.

- The educator will need to bring in space to attend to [Indicator 103.h](#). Consider using [this template](#) to offer students space to reflect and document their thinking at key points in time. Spaces where we see an opportunity for this are indicated by a triangle ▲.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one’s thinking has excellent value in solidifying one’s knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students’ identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you...
103.a: Engage in the modeling cycle (performance assessment).	i. engage with the full modeling cycle (problem, formulate, compute, interpret, validate, revise as necessary, report).
103.b: Interpret quadratic functions that arise in applications in terms of the context.	i. select two different data points that illustrate something interesting or important about the context you are modeling and describe what these points tell you about the topic.
	ii. use a model to compute output values and then interpret the values in the problem’s context.
	iii. describe the domain of the quadratic function, including anything noticed about values that are not part of the domain or why the shape of the model might change for particular values of the domain.

Content and Practice Expectations	Indicators Choose an artifact where you...
	iv. describe the contextual meaning of the average rate of change between two points for a quadratic function.
103.c: Analyze quadratic functions using different representations.	i. create a quadratic graph to show a particular context and describe important values that help understand something about the problem you are exploring.
	ii. explain how the parameters of a quadratic model relate to its graph.
103.d: Build quadratic functions that model relationships between two quantities.	i. identify the variables of interest in a given situation or data set that are interesting and that demonstrate a quadratic relationship.
	ii. build a function that models the quadratic relationship between two quantities in a situation.
103.e: Construct and compare linear and quadratic models and solve problems.	i. distinguish between situations that can be modeled with linear functions and those that can be modeled with quadratic functions, based on the quantities involved or through analysis of the rate of change.
	ii. provide insight into why some relationships are linear and others are quadratic, or explain why, in terms of a situation being modeled, a quadratic function is a better model than a linear function.
103.f: Interpret expressions for quadratic functions in terms of the situation they model.	i. describe the meaning of the parameters for a quadratic function in terms of the context.
103.g: Summarize, represent, and interpret data on two quantitative variables for linear and quadratic model fits.	i. choose a particular representation of your data to communicate something important to a particular person or audience and explain why you made that choice. In this badge, students are encouraged to investigate patterns of association in bivariate data, which includes informal description of the fit of the curve and addresses the usefulness of the model for the particular context.

Content and Practice Expectations	Indicators Choose an artifact where you...
103.h: Understand the relevance of modeling with quadratic functions.	i. recognize something that surprised you or that you learned about the topic through quadratic modeling of data. Share what surprised you or what you learned.
	ii. highlight something you are most proud of in your learning about quadratic modeling.
	iii. explain how modeling can be used to answer important questions.
103.i: Use a quadratic function model to determine values of interest in a real-world problem.	i. determine one or more output values for specific inputs to the function.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	MT
103.a.i																			X
103.b.i	X					X	X			X				X				X	X
103.b.ii	X					X	X							X				X	X
103.b.iii	★				★	X	X			★				★			X	X	
103.b.iv					X	X	X											X	
103.c.i		X			X	X	X							X			X	X	X
103.c.ii		X			X	X	X			X				X			X		X
103.d.i																			X

103.d.ii		X	X		X	X	X									X		X
103.e.i		X				X												
103.e.ii		X				X												
103.f.i					X	X	X						X			X	X	X
103.g.i																		
103.h.i	X				▲	▲	▲			▲			▲			▲	▲	▲
103.h.ii	▲				▲	▲	▲			▲			▲			▲	▲	▲
103.h.iii	▲				▲	▲	▲			▲			▲			▲	▲	▲
103.i.i	X				X	X	X						X			X	X	X

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
6.1	M Consider asking students, <i>What are some reasonable values for the length and width?</i> to bring in 103.b.iii (★). Consider using the linked template (▲) to give students access to all indicators of 103.h.	103.b.i, ii, iii 103.h.i 103.i.i	LP3, LP6, LP7	Context: Enclosing a rectangular garden producing the largest possible area Math focus: Linear, not linear, length, width, area, plotting points
6.2	K	103.c.i, ii 103.d.ii 103.e.i, ii	LP1, LP2, LP3, LP7	Context: Providing sequences of dots and boxes (visual patterns) and explaining how they change Math focus: Linear change, quadratic change, patterns, graphs, sequence, tables
6.3	K	103.d. ii	LP1, LP2, LP3	Context: Visual patterns Math focus: Quadratic patterns and equations
6.4	D			Context: Patterns

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	The CPEs do not ask students to compare quadratic and exponential functions.			Math focus: Quadratic and exponential patterns
6.5	M Consider asking students, <i>What are some reasonable values for time?</i> to bring in 103.b.iii (★). Consider using the linked template (▲) to give students access to all indicators of 103.h.	103.b.iii 103.c.i, ii 103.d.ii 103.f.i 103.i.i 103.h.i, ii, iii	LP1, LP2, LP3, LP5, LP6, LP7	Context: Rock dropping from building Math focus: Quadratic table, equation, and graph
6.6	M Consider using the linked template (▲) to give students access to all indicators of 103.h.	103.b.i, ii, iii 103.c.i, ii 103.d.ii 103.e.i, ii 103.f.i 103.h.i, ii, ii 103.i.i	LP1, LP2, LP3, LP5, LP6, LP7	Context: Shooting a cannonball into the air Math focus: Tables, graphs and equations, distance, time, height, linear and quadratic expressions, vertex, zero, domain, parabola
6.7	M Consider using the linked template (▲) to	103.b.i, ii, iii 103.c.i, ii 103.d. ii	LP1, LP2, LP3, LP5, LP6, LP7	Context: Downloading a movie, business, revenue, area and perimeter of rectangle, patterns,

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	give students access to all indicators of 103.h.	103.f.i 103.h.i, ii, ii 103.i.i		fallen object Math focus: Domain, vertex, zeroes, x-intercept
6.8	D No related Content and Practice Expectations. Compares equivalent expressions using diagrams—no context.			Context: Purely mathematical Math focus: Equivalent quadratic expressions using diagrams
6.9	D No related Content and Practice Expectations. In this lesson, students use rectangular diagrams to reason about the product of two differences or of a sum and difference and to write equivalent expressions.			Context: Purely mathematical Math focus: Standard form and factored form
6.10	M Consider asking students, <i>What are some</i>	103.b.i, iii 103.c.ii 103.h.i, ii, ii	LP1, LP2, LP3, LP7	Context: Different forms of an equation with graphs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	<p><i>reasonable values for t in this equation?</i> to bring in 103.b.iii (★). Consider using the linked template (▲) to give students access to all indicators of 103.h.</p>			<p>Math focus: Vertical and horizontal intercepts, connecting expressions with graphs</p>
6.11	<p>D Note that the context in this lesson is purely mathematical, making the connection to modeling with geometry weak.</p>			<p>Context: Purely mathematical Math focus: Graphing from factored form</p>
6.12	<p>D No related Content and Practice Expectations. Students explore the relationship between the coefficient of the squared term and the constant term.</p>			<p>Context: Purely mathematical Math focus: Identifying equivalent quadratic expressions in standard and factored form and their corresponding graph</p>

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
6.13	D Note that the context in this lesson is purely mathematical, making the connection to modeling with geometry weak.			Context: Purely mathematical Math focus: Standard and factored equations, writing equations from graphs
6.14	M Consider asking students, <i>What are some reasonable values for t in this equation?</i> to bring in 103.b.iii (★). Consider using the linked template (▲) to give students access to all indicators of 103.h.	103.b.i, ii, iii 103.c.i, ii 103.f.i 103.h.i, ii, ii 103.i.i	LP1, LP2, LP3, LP6, LP7	Context: Frog's jump, catapulting pumpkins, baseball being hit by a player, toy rocket, tennis balls Math focus: Motion of projectiles, maximum height, vertex, vertical and horizontal intercepts, initial height, zeros
6.15	D Note that the context in this lesson is purely mathematical, making the connection to modeling with geometry weak. Students do have the opportunity to			Context: Purely mathematical Math focus: Vertex form, standard form, factored form

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	analyze quadratic functions using different representations.			
6.16	D No related Content and Practice Expectations. Students think about how the structure of the equation helps to show whether the vertex of a graph represents the minimum or the maximum value of the function.			Context: Purely mathematical Math focus: Minimum, maximum, vertex, vertex form, connections to graph of quadratic function
6.17	M Consider using the linked template (▲) to give students access to all indicators of 103.h.	103.b.iii 103.c.i, ii 103.d.ii 103.f.i 103.h.i, ii, ii 103.i.i	LP1, LP2, LP3, LP5, LP6, LP7	Context: Computer animation, peanut jumping a wall, smiley face Math focus: Affects to the vertex
7.24	M Engage students in <u>Activity 24. 2</u> from Unit 7. Allow for use of	103.b.i 103.b.ii 103.b.iii 103.b.iv	LP7	Context: Height of a diver above water Math focus: Using a given equation to

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	technology. Add questions during discussion to get at 103.b.iii, 103.b.iv, and other indicators listed that are not already built in.	103.c.i 103.f.i 103.h.i 103.h.ii 103.h.iii 103.i.i		gain insights into points of interest about a context
Modeling Task- <u>Designing a Fountain</u>	M Consider using the linked template (▲) to give students access to all indicators of 103.h.	103.a.i 103.b.i, ii 103.c.i, ii 103.d.ii 103.f.i 103.h.i, ii, ii 103.i.i	LP1, LP2, LP3, LP4, LP5, LP6, LP7	Context: Decorative pool with fountain Math focus: Quadratic equations in vertex form, vertex, and height