

M153: Reasoning and Proof through Similarity

[Illustrative Mathematics–Geometry Unit 3: Similarity](#)

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4)**. This can be accomplished by learning about students’ interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum’s mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- Consider giving access to dynamic geometry software such as GeoGebra or Desmos-Geometry throughout, so that students develop proficiency in using **technology as a tool for problem-solving and understanding (LP 7)**.
- The use of [Math Language Routine 1: Stronger and Clearer Each Time](#) may be useful in providing students opportunities to revisit their initial conjectures and make them stronger. By providing students opportunities to share their conjectures with each other and with the class, students can then go back and revise their own conjectures. There are opportunities for this MLR in Lessons 3, 5, 8, 9, and 14. This is necessary to fully meet content and practice expectations [153.d](#), [153.e](#), [153.f](#), and [153.g](#).

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one’s thinking has excellent value in solidifying one’s knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students’ identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you...
153.a: Given a geometric figure and a dilation, draw the transformed figure using tools such as graph paper, tracing paper, or geometry software.	i. create a dilated figure using graph paper, tracing paper, or geometry software.
	ii. explain how you dilated a figure.
153.b: Specify a sequence of transformations that will carry a given figure onto a similar figure.	i. describe a sequence of transformations that would carry a figure onto a similar figure.
153.c: Verify experimentally the properties of dilations given by a center and a scale factor.	i. create a dilated figure using graph paper, tracing paper, or geometry software and describe the effect of dilations on line segments.
	ii. create a dilated figure using graph paper, tracing paper, or geometry software and describe the effect of dilations on angles.
153.d: Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar.	i. write an argument or explanation for why two figures are similar, using similarity transformations in your reasoning.
	ii. write an argument or explanation for why two figures are not similar, using similarity transformations in your reasoning.
	iii. use the definition of dilation within your argument.
153.e: Using similarity transformations, explain the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	i. demonstrate, using similarity transformations an example, that similar figures have corresponding angles that are equal in measure.
	ii. explain why, in general, similar figures have corresponding angles that are equal in measure, using similarity transformations in your reasoning.
	iii. demonstrate, using an example, that similar figures have corresponding sides that are proportional.

Content and Practice Expectations	Indicators Choose an artifact where you...
	iv. explain why, in general, similar figures have corresponding sides that are proportional, using similarity transformations in your reasoning.
153.f: Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	i. demonstrate, using an example, that if two figures have two corresponding angles that are equal in measure, then they are similar.
	ii. explain why, in general, two figures that have two corresponding angles that are equal in measure are similar, using similarity transformations in your reasoning.
153.g: Use similarity to prove theorems about triangles.	i. read a proof of a statement about a triangle or triangles in general and summarize or explain it in your own words.
	ii. write a proof of a statement about a triangle or triangles in general, using similarity transformations or similarity criteria in your reasoning.
153.h: Use similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	i. find an unknown length measurement in a triangle and explain how you determined your answer, using similarity transformations or similarity criteria in your reasoning.
	ii. find an unknown angle measurement in a triangle and explain how you determined your answer, using similarity transformations or similarity criteria in your reasoning.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	MT
153.a.i	X		X	X													
153.a.ii	X		X	X													

153.b.i						X											
153.c.i			X	X			X										
153.c.ii			X	X			X										
153.d.i						X	X	X				X	X				
153.d.ii					X	X	X	X					X				
153.d.iii					X		X	X									
153.e.i				X	X	X	X					X					
153.e.ii				X	X	X	X					X					
153.e.iii			X	X	X		X					X					
153.e.iv			X		X		X					X					
153.f.i									X		X						X
153.f.ii									X		X						X
153.g.i								X					X				
153.g.ii					X			X	X		X		X	X			
153.h.i					X						X		X		X	X	X
153.h.ii									X							X	

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority e

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.1	K	153.a.i 153.a.ii	LP2, LP7	Context: Builds on student understanding of images, using common knowledge of scaled and distorted images Math focus: Scale drawings, dilations
3.2	D Listed as an optional lesson by authors. This lesson serves both as practice with proportional relationships and extension into an interesting context. Students also have the opportunity to work with large numbers and consider what they mean—how vast is space, really?			Context: Scale drawing of the planets Math focus: Creating and interpreting scaled drawings of figures
3.3	K Consider asking students to describe the effect of the dilations on the angles within this lesson to address CPE 153.c.ii.	153.a.i 153.a.ii 153.c.i 153.c.ii 153.e.iii 153.e.iv	LP2, LP4, LP7	Context: All mathematical context; no real life context used Math focus: Creating dilations given a scale factor and looking at the effect on side lengths

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.4	K	153.a.i 153.a.ii 153.c.i 153.c.ii 153.e.i 153.e.ii 153.e.iii	LP7	Context: All mathematical context; no real life context used Math focus: Verifying experimentally and asserting that dilations take angles to congruent angles; using this assertion to create a convincing argument that if dilations take angles to congruent angles, they must take lines to parallel lines
3.5	K Consider asking students to describe the relationship between corresponding angles within this lesson to fully address CPE 153.e.	153.d.ii 153.d.iii 153.e.i 153.e.ii 153.e.iii 153.e.iv 153.g.ii 153.h.i	LP1, LP3	Context: All mathematical context; no real life context used Math focus: Using the definition of dilation to prove that the triangles formed by connecting midpoints of two sides are dilations of the original triangle
3.6	K	153.b.i 153.d.i 153.d.ii 153.e.i 153.e.ii	LP3, LP4	Context: All mathematical context; no real life context used Math focus: Connecting similarity with transformations
3.7	K	153.c.i 153.c.ii 153.d.i 153.d.ii 153.d.iii	LP1, LP3	Context: All mathematical context; no real life context used Math focus: Using rigid transformations and dilations to reason about and make

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		153.e.i 153.e.ii 153.e.iii 153.e.iv		generalizations about similarity of triangles and other types of figures
3.8	K	153.d.i 153.d.ii 153.d.iii 153.g.i 153.g.ii	LP2, LP3	Context: All mathematical context; no real life context used Math focus: Proving figures to be similar or not similar
3.9	K	153.f.i 153.f.ii 153.g.ii 153.h.ii	LP3, LP5	Context: All mathematical context; no real life context used Math focus: Proving triangles are similar through the AA criterion
3.10	D Listed as an optional lesson by authors. No connection to CPEs.			Context: All mathematical context; no real life context used Math focus: Connection between SSS triangle congruence theorem and SSS similarity theorem, as well as connection between SAS triangle congruence theorem and SAS triangle similarity theorem
3.11	K	153.f.i 153.f.ii 153.g.ii 153.h.i	LP2, LP3, LP7	Context: All mathematical context; no real life context used Math focus: Parallel segments in triangles,

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
				finding the missing side lengths
3.12	D Listed as an optional lesson by authors. If choose to use, consider Amplifying Activity 12.3 to give students opportunity with 153.h.i. Lesson previews work with Pythagorean theorem and practice finding unknown values in proportional relationships. Consider Amplifying Activity 12.3.			Context: Garden spaces and quilts Math focus: Calculating unknown values in proportional relationships and finding scale factors
3.13	K	153.d.i 153.e.i 153.e.ii 153.e.iii 153.e.iv 153.g.ii 153.h.i	LP1, LP2, LP3, LP5	Context: All mathematical context; no real life context used Math focus: Using Pythagorean theorem and similarity to solve missing side lengths
3.14	K	153.d.i 153.d.ii 153.g.i 153.g.ii	LP3, LP5	Context: All mathematical context; no real life context used Math focus: Proving the Pythagorean theorem using similar triangles

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.15	M Prioritize 15.1 and 15.2 to focus on finding unknown lengths of a triangle and explaining the use of similarity criteria.	153.h.i	LP3, LP4	Context: All mathematical context; no real life context used Math focus: Finding unknown lengths of similar right triangles
3.16	K Consider asking students to describe the relationship between corresponding angles within this lesson to fully address CPE 153.h.	153.f.i 153.f.ii 153.h.i 153.h.ii	LP1, LP3, LP5, LP6	Context: Shooting pool, measuring tall objects with mirrors Math focus: Using similarity to solve problems
Modeling Task - Scaling a Playground	K	153.h.i	LP1, LP2, LP3, LP4, LP6	Context: Tracing the path of a cup rolling on a flat surface Math focus: Applying theorems about circles, using geometric shapes and their measures/properties to describe objects, using congruence and similarity criteria for triangles to solve problems