

M152: Reasoning and Proof through Congruence

Illustrative Mathematics–Geometry Unit 2-Congruence

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4)**. This can be accomplished by learning about students’ interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum’s mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum and opportunities to use the embedded math language routines to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- Educators will need to bring in space to attend to Indicator 152.a.i. Consider using Geometry Unit 1 Lessons 10, 13-14 to offer students space to use these skills. Spaces where we see an opportunity for this are indicated by a square ■.
- Educators will need to bring in space to attend to Indicator 152.j.i and 152.j.ii. Consider using Geometry Unit 1 Lesson 5 to offer students space to use these skills. Spaces where we see an opportunity for this are indicated by a triangle ▲.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one’s thinking has excellent value in solidifying one’s knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students’ identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you...
152.a: Use definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	i. explain what a rotation, reflection, or translation is using appropriate terms: angle, circle, perpendicular line, parallel line, or line segment.
152.b: Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using tools such as graph paper, tracing paper, or geometry software.	i. create a rotated, reflected, and/or translated figure using graph paper, tracing paper, or geometry software.
	ii. explain how you rotated, reflected, and/or translated a figure.
152.c: Specify a sequence of transformations that will carry a given figure onto another.	i. describe a sequence of transformations that would carry a figure onto another figure.
152.d: Given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	i. write an argument or explanation for why two figures are congruent, using similarity transformations in your reasoning.
	ii. write an argument or explanation for why two figures are not congruent, using similarity transformations in your reasoning.
152.e: Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	i. demonstrate using an example that if two triangles are congruent, then corresponding angles and corresponding sides are also congruent.
	ii. explain why, in general, if two triangles are congruent, then corresponding angles and corresponding sides are also congruent.
	iii. demonstrate using an example that if two triangles have corresponding angles and corresponding sides that are congruent, then the two triangles are also congruent.

Content and Practice Expectations	Indicators Choose an artifact where you...
	iv. explain why, in general, if two triangles have corresponding angles and corresponding sides that are congruent, then the two triangles are also congruent.
152.f: Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	i. demonstrate using an example that if two triangles meet one of the triangle congruence criteria (ASA, SAS, or SSS), then they are congruent according to the definition of congruence in terms of rigid motions.
	ii. explain why, in general, if two triangles meet one of the triangle congruence criteria (ASA, SAS, or SSS), then they are congruent according to the definition of congruence in terms of rigid motions.
152.g: Prove theorems about lines and angles.	i. find a missing length or angle measurement in a diagram containing lines and angles and explain how you determined your answer.
	ii. read a proof of a statement about lines and angles (a specific case or in general) and summarize or explain it in your own words.
	iii. write a proof of a statement about lines and angles (a specific case or in general), using rigid transformations or the concept of congruence in your reasoning.
152.h: Prove theorems about triangles.	i. find a missing length or angle measurement in a triangle and explain how you determined your answer.
	ii. read a proof of a statement about a triangle or triangles in general and summarize or explain it in other words.
	iii. write a proof of a statement about a triangle or triangles in general, using rigid transformations or the concept of congruence in your reasoning.
152.i: Prove theorems about parallelograms.	i. find a missing length or angle measurement in a parallelogram and explain how you determined your answer.
	ii. read a proof of a statement about a parallelogram or parallelograms in general and

Content and Practice Expectations	Indicators Choose an artifact where you...
	summarize or explain it in your own words.
	iii. write a proof of a statement about a parallelogram or parallelograms in general, using rigid transformations or the concept of congruence in your reasoning.
152.j: Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).	i. construct a geometric figure using compass and straightedge, dynamic geometric software, or other tools.
	ii. explain how you constructed a geometric figure.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	MT
152.a.i	■	■	■													
152.b.i	X	X				X										
152.b.ii					X	X			X							
152.c.i	X	X			X	X			X							
152.d.i	X	X	X		X	X			X							X
152.d.ii		X	X													X
152.e.i	X			X												
152.e.ii	X			X										X		

152.e.iii	X	X														
152.e.iv	X	X		X										X		
152.f.i						X	X		X	X						
152.f.ii						X	X		X	X						
152.g.i					X			X								
152.g.ii					X			X						X		
152.g.iii					X			X								
152.h.i						X			X							
152.h.ii						X	X	X	X	X				X		
152.h.iii						X	X	X	X					X		
152.i.i													X			
152.i.ii									X			X	X			
152.i.iii									X			X	X		X	
152.j.i								▲						▲		
152.j.ii								▲						▲		

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.1 Add Unit 1 Lesson 10 Activities as described	M Two days to complete. Recommended sequence of activities: Unit 1 Lesson 10 Activity 1 (1.10.1) and Unit 1 Lesson 10 Activity 2 (1.10.2) to introduce language of transformations, reflections, and rotations. Skip Unit 2 Lesson 1 Activity 1. Do Activity 2.1.2 and Activity 2.1.3	152.a.i 152.b.i 152.c.i 152.d.i 152.e.i 152.e.ii 152.e.iii 152.e.iv	LP1, LP2	Context: All mathematical context; no real life context used Math focus: Corresponding sides and angles
2.2 Add Unit 1 Lesson 13 Activities as described	M Recommended sequence of activities: Use 1.13.1 and 1.13.2 to focus on students using appropriate language to write a sequence of rotations and create a sequence of rotations. Complete Unit 2 Lesson 2 Activity 1 (2.2.1) and Activity 2.2.2. Skip Activity 2.2.3 if	152.a.i 152.b.i 152.c.i 152.d.i 152.d.ii 152.e.iii 152.e.iv	LP3	Context: All mathematical context; no real life context used Math focus: Corresponding parts

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	needed.			
2.3 Add Unit 1 Lesson 14 Activities as described	M Recommended sequence of activities: Use 1.14.3 to give students an opportunity to create triangles using rotations. Use 2.3.2 to focus on corresponding parts of triangles. Skip Activities 2.3.1 and 2.3.3 if needed	152.a.i 152.d.i 152.d.ii	LP2, LP3	Context: All mathematical context; no real life context used Math focus: Congruent triangles
2.4	K	152.e.i 152.e.ii 152.e.iv	LP1, LP2	Context: All mathematical context; no real life context used Math focus: Congruent triangles
2.5	K As prompts permit, ask students about measures of corresponding sides and angles to meet CPE 152.g.i.	152.b.ii 152.c.i 152.d.i 152.g.i 152.g.ii 152.g.iii	LP2	Context: All mathematical context; no real life context used Math focus: Congruent segments
2.6	K	152.b.i 152.b.ii 152.c.i 152.d.i	LP2, LP3, LP7	Context: All mathematical context; no real life context used Math focus: Side-angle-side triangle

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		152.f.i 152.f.ii 152.h.i 152.h.ii 152.h.iii		congruence
2.7	K	152.f.i 152.f.ii 152.h.ii 152.h.iii	LP2, LP3, LP7	Context: All mathematical context; no real life context used Math focus: Angle-side-angle triangle congruence
2.8 Add Unit 1 Lesson 5 Activities as described	M Recommended sequence of activities: Use 2.8.1 to warm up students. Use 1.5.1 and 1.5.2 to give students an opportunity to construct perpendicular bisectors, followed by 2.8.2 and 2.8.3. As prompts permit, ask students about measures of corresponding sides and angles to meet CPE 152.g.i.	152.g.i 152.g.ii 152.g.iii 152.h.ii 152.h.iii 152.j.i 152.j.ii	LP1, LP2, LP7	Context: All mathematical context; no real life context used Math focus: Perpendicular bisector theorem

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2.9	K	152.b.ii 152.c.i 152.d.i 152.f.i 152.f.ii 152.h.i 152.h.ii 152.h.iii 152.i.iii	LP1, LP3, LP7	Context: All mathematical context; no real life context used Math focus: Side-side-side triangle congruence, parallelograms
2.10	M Prioritize Activity 10.2 to focus on SAS, ASA, and SSS triangle congruence theorem. Skip Activity 10.3 if needed.	152.f.i 152.f.ii 152.h.ii 152.i.ii	LP2	Context: All mathematical context; no real life context used Math focus: Proofs, congruence
2.11	D Not needed to enter the next lesson. Lesson marked as optional by authors.			Context: All mathematical context; no real life context used Math focus: Students study ambiguous case of S-S-A triangle congruence
2.12	K	152.i.ii 152.i.iii	LP2	Context: All mathematical context; no real life context used Math focus: Parallelogram proofs
2.13	K	152.i.i 152.i.ii	LP1, LP2	Context: All mathematical context; no real life context used

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		152.i.iii		Math focus: Parallelogram proofs
2.14 Add Unit 1 Lesson 5 Activities as described	M Recommended sequence of activities: Use 1.5.3 to give students an opportunity to create an angle bisector. Follow with 2.14.1 and 2.14.2. Skip 2.14.3 if needed.	152.e.ii 152.e.iv 152.g.ii 152.h.ii 152.h.iii 152.j.i 152.j.ii	LP1, LP3	Context: All mathematical context; no real life context used Math focus: Proofs with construction
2.15	K	152.i.iii	LP1, LP3, LP4, LP5	Context: All mathematical context; no real life context used Math focus: Students apply the triangle congruence theorems to parallelograms in order to generate and prove congruence theorems for parallelograms
Modeling Task- <u>How Much Water?</u>	M Introduce the idea of congruence when students are brainstorming ideas of different types of containers. Have students ponder the concept of scaling and how that relates to congruence.	152.d.i 152.d.ii	LP4, LP5, LP6	Context: Things we use that consume water Math focus: Volume, dilations, defining appropriate quantities for descriptive modeling