

# M204: Exponential and Logarithmic Functions and Equations

## [Illustrative Mathematics–Algebra 2 Unit 4: Exponential Functions and Equations](#)

### Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4)**. This can be accomplished by learning about students’ interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum’s mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum, such as math language routines, to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.

### Learning Principles

**Engage with cognitively demanding tasks in heterogeneous settings (LP 1).** Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

**Engage in social activities (LP 2).** Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one’s thinking has excellent value in solidifying one’s knowledge.

**Build conceptual understanding through reasoning (LP 3).** Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

**Have agency in their learning (LP 4).** Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

**View mathematics as a human endeavor across centuries (LP 5).** Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

**See mathematics as relevant (LP 6).** Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

**Employ technology as a tool for problem-solving and understanding (LP 7).** Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

## Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you...
204.a: Understand the relationship between exponential and logarithmic functions.	i. use reasoning about exponents to explain the value of a logarithmic expression.
	ii. write an exponential expression to verify the value of a logarithmic expression.
204.b: Graph exponential and logarithmic functions.	i. graph an exponential function over a domain that includes non-integers.
	ii. graph a logarithmic function.
	iii. explain the effect of changing the parameters in an exponential or logarithmic function on its graph.
204.c: Interpret logarithmic and exponential functions that arise in applications in terms of the context.	i. interpret key features of the graph of an exponential function in terms of the context being modeled.
	ii. interpret key features of the graph of a logarithmic function in terms of the context being modeled.
	iii. explain the choice of domain for a logarithmic or exponential function in terms of the context being modeled.
	iv. calculate and explain the meaning of the average rate of change for an exponential or logarithmic function in terms of the context being modeled.
204.d: Use logarithms to analyze exponential models.	i. create an exponential model for a context.
	ii. use a logarithm to solve for an unknown value of an exponential model.
	iii. interpret values for exponential models in context.

## Unit Overview

	<u>L1</u>	<u>L2</u>	<u>L3</u>	<u>L4</u>	<u>L5</u>	<u>L6</u>	<u>L7</u>	<u>L8</u>	<u>L9</u>	<u>L10</u>	<u>L11</u>	<u>L12</u>	<u>L13</u>	<u>L14</u>	<u>L15</u>	<u>L16</u>	<u>L17</u>	<u>L18</u>	<u>MT</u>
204.a.i									X	X	X						X		
204.a.ii									X	X	X			X			X		
204.b.i			X	X	X	X	X	X											X
204.b.ii															X		X	X	X
204.b.iii					X	X							X			X	X		
204.c.i			X	X	X	X						X	X		X	X			X
204.c.ii															X		X	X	
204.c.iii			X	X	X												X	X	X
204.c.iv			X	X	X	X	X	X											X
204.d.i	X	X	X	X	X	X	X	X				X	X						
204.d.ii										X	X			X	X				
204.d.iii		X	X	X		X	X						X		X	X			

Note: Columns in gray indicate a recommendation to delete the lesson

## Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
4.1	K The content lays a foundation for identified CPEs.	204.d.i	LP1, LP2, LP3, LP6	<b>Context:</b> Shrinking a passport photo; pond in a park  <b>Math focus:</b> Calculating values that are changing exponentially
4.2	K Listed as an optional lesson by authors, but a good opportunity to address CPEs listed. Consider asking students for the exponential model in Activity 2.3 for stronger connection to CPE 204.d.i.	204.d.i 204.d.iii	LP1, LP2, LP3, LP6	<b>Context:</b> Paying Grandma back; climbing cost of college tuition; decreasing value of two vans  <b>Math focus:</b> Seeing that exponential functions change by equal factors over equal intervals
4.3	M Leverage Activity 3.2 to fully address CPE 204.b and CPE 204.c by explicitly asking students about reasonable domains	204.b.i 204.c.i 204.c.iii 204.c.iv 204.d.i 204.d.iii	LP1, LP2, LP3, LP6, LP7	<b>Context:</b> Population growth in Florida; disappearing medicine  <b>Math focus:</b> Determining the value of exponential functions at non-whole number inputs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	for the functions used and connections between the function, graph, and context.			
4.4	K	204.b.i 204.c.i 204.c.iii 204.c.iv 204.d.i 204.d.iii	LP1, LP3, LP6, LP7	<b>Context:</b> Population growth in Nigeria; half-life of caffeine  <b>Math focus:</b> Calculating a growth or decay factor of an exponential function for different input values
4.5	M Leverage activities to address CPE 204.b and CPE 204.c by explicitly asking students about reasonable domains for the functions used and connections between the function, graph, and context.	204.b.i 204.b.iii 204.c.i 204.c.iii 204.c.iv 204.d.i	LP1, LP2, LP3, LP7	<b>Context:</b> Machine depreciation; fever medicine concentration in body  <b>Math focus:</b> Explaining why exponential functions change by the same factor over equal intervals, even when the intervals are not whole numbers
4.6	K	204.b.i 204.b.iii 204.c.i 204.c.iv	LP1, LP2, LP3	<b>Context:</b> Bacteria growth expressions  <b>Math focus:</b> Writing equations for exponential functions from two

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		204.d.i 204.d.iii		input-output pairs, even when the input pairs are not one unit apart
4.7	K	204.b.i 204.c.iv 204.d.i 204.d.iii	LP1, LP2, LP3, LP5, LP6, LP7	<b>Context:</b> Radiocarbon dating  <b>Math focus:</b> Using the half-life of elements to calculate how much of the element remains over time
4.8	K	204.b.i 204.c.iv 204.d.i 204.d.iii	LP1, LP2, LP3, LP6, LP7	<b>Context:</b> Bacteria successively splitting in a petri dish  <b>Math focus:</b> Approximating the value of unknown exponents
4.9	K	204.a.i 204.a.ii	LP1, LP2, LP3	<b>Context:</b> Purely mathematical  <b>Math focus:</b> Understanding that a logarithm is a way to represent an exponent in an exponential equation
4.10	K	204.a.i 204.a.ii 204.d.ii	LP1, LP2, LP3	<b>Context:</b> Purely mathematical  <b>Math focus:</b> Evaluating logarithms

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
4.11	K	204.a.i 204.a.ii 204.d.ii	LP1, LP2, LP3, LP7	<b>Context:</b> Purely mathematical  <b>Math focus:</b> Calculating the value of a logarithmic expression using estimation or technology
4.12	K	204.c.i 204.d.i	LP1, LP2, LP3, LP5, LP7	<b>Context:</b> Moldy growth  <b>Math focus:</b> Understanding the number $e$ is an irrational constant
4.13	K	204.b.iii 204.c.i 204.d.i 204.d.iii	LP1, LP2, LP3, LP6, LP7	<b>Context:</b> Population of a colony of insects  <b>Math focus:</b> Understanding that $e$ is used in exponential models when the growth rate is applied at every moment
4.14	K	204.a.ii 204.d.ii	LP1, LP2, LP3, LP7	<b>Context:</b> Purely mathematical  <b>Math focus:</b> Using base 10 and natural logarithms to solve exponential equations
4.15	M Amplify Activity 15.3 to introduce the graph of a logarithmic function and discuss	204.b.ii 204.c.i 204.c.ii 204.d.ii 204.d.iii	LP1, LP2, LP3, LP6, LP7	<b>Context:</b> Retire a millionaire; cicada population  <b>Math focus:</b> Solving exponential equations using graphs or logs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	key features of the graph to attend to CPE 204.b.ii and 204.c.ii.			
4.16	K	204.b.iii 204.c.i 204.d.iii	LP1, LP2, LP3, LP6	<b>Context:</b> Two bank accounts; bacteria in different conditions; populations of two countries  <b>Math focus:</b> Calculating and interpreting the intersection of the graphs of two exponential functions
4.17	K	204.a.i 204.a.ii 204.b.ii 204.b.iii 204.c.ii 204.c.iii	LP1, LP2, LP3, LP6, LP7	<b>Context:</b> Bacteria growth—doubling every hour; growing by a factor of 10 every day  <b>Math focus:</b> Interpreting logarithmic functions in context
4.18	M Lesson listed as optional by the authors, but a good opportunity to address CPE connected to logarithmic functions as listed. Consider	204.b.ii 204.c.ii 204.c.iii	LP1, LP2, LP3, LP7	<b>Context:</b> How acidic is it; pH ratings; measuring earthquake strength  <b>Math focus:</b> Understanding how logarithms are used to measure acidity and the intensity of earthquakes

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	asking questions not present in curricular notes to address the full scope of listed indicators.			
Modeling Task- <u>Exponential Situations</u>	M Consider using the situations offered as a menu of topics from which students can conduct research, gather data, and analyze for appropriate mathematical models to broaden the opportunity to engage in LPs listed.	204.b.i 204.b.ii 204.c.i 204.c.iii 204.c.iv	LP1, LP2, LP3, LP4, LP5, LP6, LP7	<p><b>Context:</b> Number of cars in a country; population changes; bacteria population; carbon dating; half-life; changes in quantity of books in a library</p> <p><b>Math focus:</b> Asking and answering questions about a situation that can be modeled with an exponential function</p>