

M111B Performance Task

Measuring Up

The Modeling Cycle

In this task, you will examine statistics associated with a sports champion, a sleep study of Americans, or another topic of your choice to compare to your own analysis of a smaller, local population and apply the basic mathematical modeling cycle to make predictions, interpret information, and report your findings.

Mathematical models are used to make sense of ideas and phenomena in our world. As illustrated in the diagram below, the modeling cycle is a process of collecting data about an idea or problem, creating models of that data, analyzing the story told by those models, and reporting the results to the appropriate audience.

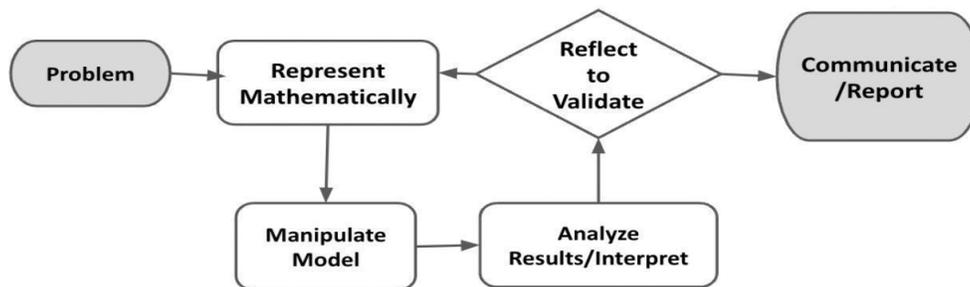


Figure 1. *Diagram of Modeling Process.* From “Progressions for the Common Core State Standards for Mathematics” [Draft manuscript], by CCSS Writing Team, 2019, p. 6.

In this task, you will be engaging with this mathematical modeling cycle to analyze data, evaluate findings, and identify trends. You will use the models and your takeaways from this task to create a product that reflects your interpretation of your data. You will generate and describe statistics such as standard deviation and median, create and justify a visual to explain your data, and identify an audience who would benefit from your data, among other final questions and tasks.

Introduction: Measuring Up

We compare a lot as a society. Who is the best actor? Which car is the better buy? How much does the typical American spend on toilet paper? It is how we determine the best, worst, and typical of everything in populations of people, places, and things. Any time we collect numerical data for one variable, we are analyzing one-variable statistics.

QUESTION 1



Think about yourself. Make a list of things you enjoy and/or interesting things that are happening in your life. Consider facts, interests, and/or talents.

[This question is not meant to be scored. Look for reasonable responses (see teacher-facing guide for ideas). Expect students to answer individually on their own forms, as well as participate in a small group or class discussion.]

Student A

- .Playing the horn
- .Fishing
- .Hunting
- .Dogs
- .Sundays
- .Cooking

The list provided by the student is acceptable. This introductory question is meant to help students begin to think about things they will later try to quantify.

Student B

- I play football
- Going to the movies
- Riding rides
- Driving

The list doesn't have to be long. This response is acceptable.

Student C

Some things I am interested in are volleyball, painting, weightlifting, etc. Some things I do outside of school is ASL online, some other things I like to do are hangout with friends, family and watch movies and play board games.

While this response seems to be more descriptive than Student A's or Student B's response, it doesn't make it "better" than the other two responses.

QUESTION 2



Considering what you shared in Question 1, do you think you do more, do less, are better, or are worse? Or are you average, in the middle?

What quantities could be measured to make a comparison? Why would a comparison be important?

Student A

I believe i known more about Horror media and movies then the average person
I believe i do better than most people at Taekwondo
I believe im average at reading as i read 1-2 books within a few weeks
I believe i could do better during Latin class and studies
I believe that i'm in the middle of knowing Greek and Roman mythology as i known more then people who don't learn about it and less then people who have taken those classes

I could measure how many horror movies I've watched in a year or how many medals I've won at taekwondo tournaments. Comparisons are important in order to see how i could improve or how well I'm doing.

This student detailed how they compare to each of their responses in Question 1 and thoroughly answered which quantities could be measured and why a comparison is important to them.

Student B

I consider myself above average in those things. For football I play in every game for the gym i can lift more than a lot of people in my grade and my age. The comparison is important to see who's better and to rank people.

This student did indicate a couple quantities which could be measured to make a comparison and he did reference why making a comparison is important to him. Teachers don't need all responses to be as detailed as Student A's response. This response is adequate.

Student C

I do better. If I was scared I wouldn't be cheering and getting thrown up in the air. When I go to the beach I feel peace and happiness when swimming.

This student is referencing their answer in Question 1 related to cheerleading when they state, "I do better." What is lacking in this response is any quantities which could be measured and why comparing those quantities might be important.

Measuring Up

Choose Your Own Adventure

In this performance assessment, you will compare statistics already compiled by a group like the U.S. Census, ESPN, or another source related to the population of your choosing. You will need to gather data from an appropriate sample of your population, compute statistical measures, build models like dot plots, and compare these to pre-prepared statistics.

Your Task: Pick one of the options below:

OPTION C: Pick another topic for which you will compile a data set about yourself or something of interest to you locally to compare against statistics that have already been compiled for another population.

In the **Problem** and **Formulate** sections, you will gather data for your option, then evaluate this relationship to identify a model of best fit and create a related equation. The **Problem Formulation Checklist** can be used to ensure you have the necessary components before moving on to the remaining parts of the task.

Note: You are welcome to use additional paper or digital forms to document your work. Be sure to include all of your work in your final submission.

Problem

ACTION 1



Use search engines to help you explain why your selected comparison matters. Summarize the following information:

- State your option and what you intend to compare.
- Provide a purpose to your option.
- Describe what you expect to learn.

Student A

- State your option and what you intend to compare.

I am comparing the number of books i've read in a year

- Provide a purpose to your option.

I'd like to determine if how many books you read depicts your vocabulary usage and your behavior, along with their views on certain topics.

- Describe what you expect to learn.

I expect to determine the impact that reading has on a readers and how it affects their worldview.

This student provided answers to each question, but the teacher might want to encourage the student to answer in more detail. The student might need to focus their thoughts and ideas a little more. They mentioned books read in a year and how it affects vocabulary, behavior, and world view.

Student B

- State your option and what you intend to compare.
Average number of fish caught, fish weight, and what season they were caught during each season.

- Provide a purpose to your option.

The purpose is to determine the activity of fish during the different seasons to find the breeding months and the best times to fish.

- Describe what you expect to learn.

I expect to learn the best time of year to fish.

Similar to Student A, this student might need support narrowing ideas down. The student has discussed an interest in investigating the number of fish caught, fish weight, season best for fishing, and breeding months.

Student C

- State your option and what you intend to compare.

I am comparing me as a cheerleader skills to a Alabama cheerleader skills

- Provide a purpose to your option.

To help you strive to become a better cheerleader and see them as inspiration

- Describe what you expect to learn.

I expect to learn greater skills and a greater person to become like she is.

The student might need support clarifying her ideas and narrowing down quantifiable variables.

ACTION 2

Write a one-variable statistical question to collect data based on your response in Action 1.



[Teachers should spend time with each student at this point to ensure each student has a researchable question with varied and quantifiable data. Students should also be encouraged to think about what sample population they will ask. Three typical responses are shown below.]

Student A

How many books are read in 1 year?

Student B

How many hours a week does someone practice playing the violin on average.

Student C

How many times does it take you to perfect a skill you just learned?

ACTION 3



Define the variable you will investigate through your statistical question and the population that you will sample.

Student A

Variable- number of books read in a year
Teenagers- aged 14-17

The student adequately responded by identifying both the variable and sample population.

Student B

The amount of time that people spend practicing the violin at the ages of 14-16

The student has identified both the variable and sample population; however, the student may want to be more specific with their sample population. If the student asks a large group of students who don't play the violin, it might skew the data with 0s. Is the student asking students in their violin class? If so, that should be specified in the sample population.

Student C

My variable I will investigate is the number of tries it takes you.

The variable is not specific or measurable. Sample population was not identified.

Formulate

ACTION 4



Describe your data collection plan. Refer to these guiding questions:

- Can you investigate your statistical question? Or do you need to narrow your focus to make your question more specific?
- What data can help you investigate your statistical question?
- How much data should you collect?
- How will you collect your data?
- What type of bias might you encounter?
- How can you minimize bias?
- How frequently do you need to collect data?

Student A

- Can you investigate your statistical question? Or do you need to narrow your focus to make your question more specific? **yes**
- What data can help you investigate your statistical question? **How many hours people spend practicing the violin**
- How much data should you collect? **I should ask around 7 people**
- How will you collect your data? **By asking people in my class**
- What type of bias might you encounter? **Some of them are my friends**
- How can you minimize bias? **By asking people who aren't my friends**
- How frequently do you need to collect data? **Once a week**

The student question was, "How many hours a week does someone practice playing the violin on average?". The teacher may want to pose questions such as "What if none of your friends play violin?" or "What will your data look like?" or "Is a sample size of 7 large enough to minimize potential bias?"

Student B

- Can you investigate your statistical question? Or do you need to narrow your focus to make your question more specific? *I can investigate as the question is clear*
- What data can help you investigate your statistical question? *How many books people read in a year*
- How much data should you collect? *At least 1 person per age*
- How will you collect your data? *By talking to people and asking them while taking notes*
- What type of bias might you encounter? *Only asking people i want to ask*
- How can you minimize bias? *Ask strangers*
- How frequently do you need to collect data? *Before friday*

The student question was, "How many books are read in 1 year". The teacher may want to pose questions such as "What if others don't remember the number of books they read this year?" or "Are you including books read for school?" or "Do magazines or graphic novels count?" The student should clarify their sample size. "At least 1 person per age" Is this everyone aged 0-100 or just high school students? Is this a large enough sample size to minimize bias?

Student C

- Can you investigate your statistical question? Or do you need to narrow your focus to make your question more specific?
- What data can help you investigate your statistical question? *graphing n timing*
- How much data should you collect? *5*
- How will you collect your data? *Watching the cheerleader*
- What type of bias might you encounter? *Girls*
- How can you minimize bias? *Ask everyone*
- How frequently do you need to collect data? *Daily*

The student question was, "How many times does it take you to perfect a skill you just learned?" The teacher should revisit the first question with the student, asking the student to elaborate in more detail.

Problem Formulation Checklist

Requirement	Fulfilled?
I clearly identified what I will be comparing and its significance.	<input type="checkbox"/>
I wrote a measurable, one-variable statistical question.	<input type="checkbox"/>
I defined my variables and sample population.	<input type="checkbox"/>
I described a clear data collection plan that addresses bias.	<input type="checkbox"/>
Teacher Consultation:	_____

In the **Represent Mathematically** and **Interpret & Analyze** sections, you will collect your data, compute statistical measures of center and spread, and model the data with an appropriate visual display. Write a statement explaining why you chose this particular display. The **Represent Mathematically Checklist** can be used to ensure you have the necessary components before moving on to the remaining parts of the task.

Represent Mathematically

ACTION 5



Carry out your data collection plan. Describe any modifications, if any, you needed when carrying out your data collection plan.

[Most students just reported their data and didn't describe any modifications they may have needed to perform. It is acceptable to just see data at this point, as long as it aligns to their identified question in Action 2. If it doesn't align, the teacher should ask for clarification as to why they modified their data collection plan.]

Student A

5, 5, 10, 5, 5, 9, 6, 12

The student question was, "How many hours a week does someone practice playing the violin on average". This is an adequate response.

Student B

numbers of book read per person

1,1,2,3,2,0,10,13,1,5,0,365,9,23,2,8,2

This is an adequate response.

Student C

5,3,4,10,15,20,13,14,18

The student question was, "How many times does it take you to perfect a skill you just learned?" This is an adequate response.

ACTION 6



Generate the following statistics for your data set.

- mean
- standard deviation
- median
- interquartile range

Explain the meaning of these statistics in the context of your data set.

[All three students found appropriate statistics for their data sets but failed to explain the meaning of these statistics. They also neglected to determine the standard deviation for their data set. The teacher made need to emphasize that the interpretation of these statistics is part of the task. However, it is acceptable if students provide these explanations in some of the remaining actions/questions.]

Student A

Mean \bar{x}	7.125
Median \bar{x}	5.5
Mode	5
Range	7
Minimum	5
Maximum	12
Count n	8
Sum	57
Quartiles	Quartiles: Q ₁ --> 5 Q ₂ --> 5.5 Q ₃ --> 9.5
Interquartile Range IQR	4.5
Outliers	none

Student B

Mean 23.710526315789
Median 2
Mode 2
Range 365
Minimum 0
Maximum 365
Count n 19
Sum 450.5
Quartiles
Quartiles:
Q₁ --> 1
Q₂ --> 2
Q₃ --> 9

Student C

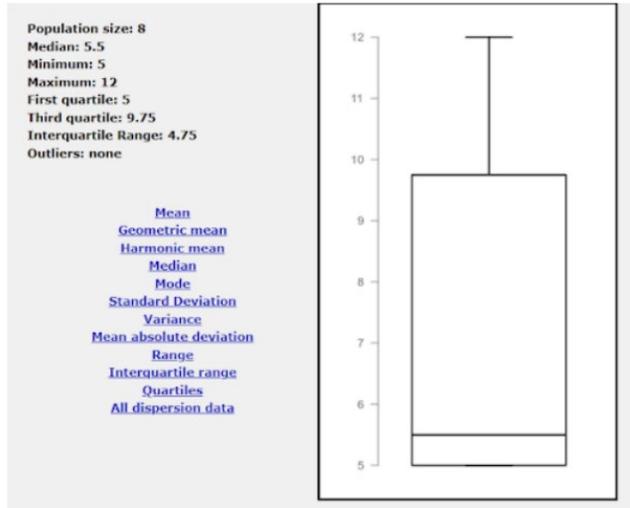
Mean - 11.3333333333333
Median - 13
Mode - 5, 3, 4, 10, 15, 20, 13, 14, 18
Range - 17
Minimum - 3
Maximum - 20
Count n - 9
Sum - 102
Quartiles
Quartiles:
Q₁ --> 4.5
Q₂ --> 13
Q₃ --> 16.5
InterquartileRange IQR 12
Outliers - none

ACTION 7

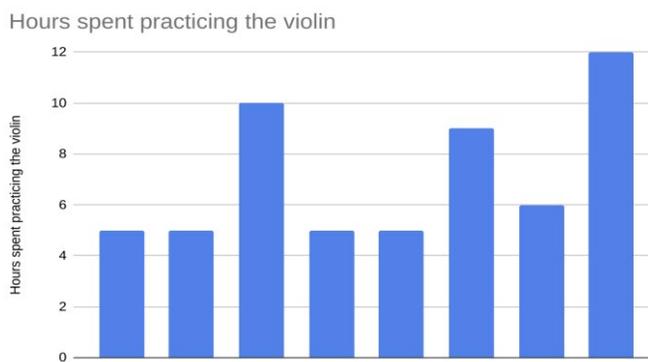
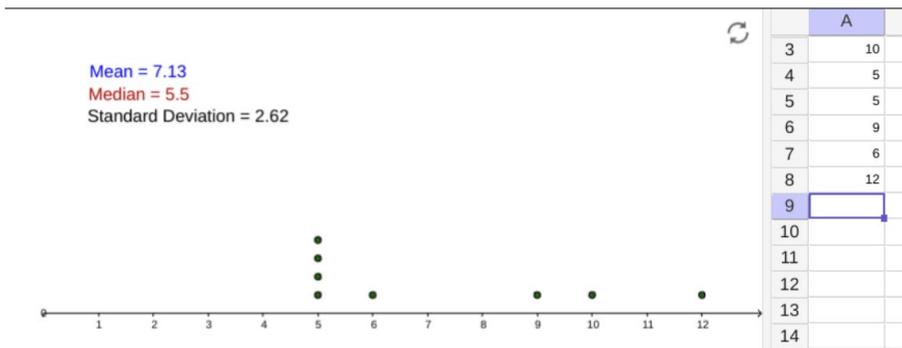


Choose and create an appropriate visual to display your data. Explain why the visual you chose is appropriate for your data.

Student A

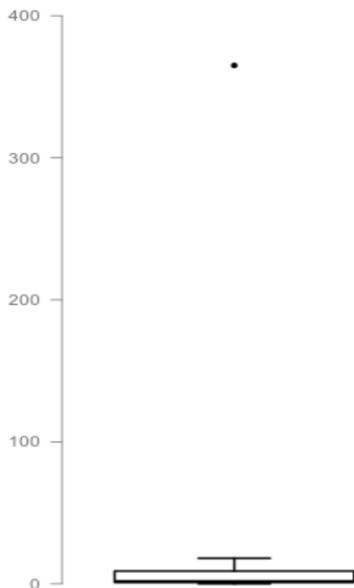


The student question was, "How many hours a week does someone practice playing the violin on average".



The student created three visual displays but did not explain why their visual representations are appropriate. Asking questions such as "What does this visual representation tell us about the data?", "What are the pros and cons of each representation?", or "What do you want your visual representation to show?" might help the student provide an adequate explanation.

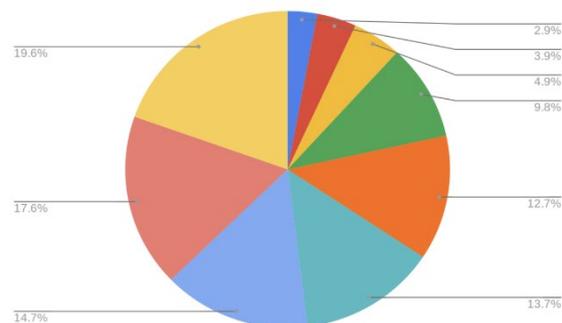
Student B



The student question was, "How many books are read in 1 year?"

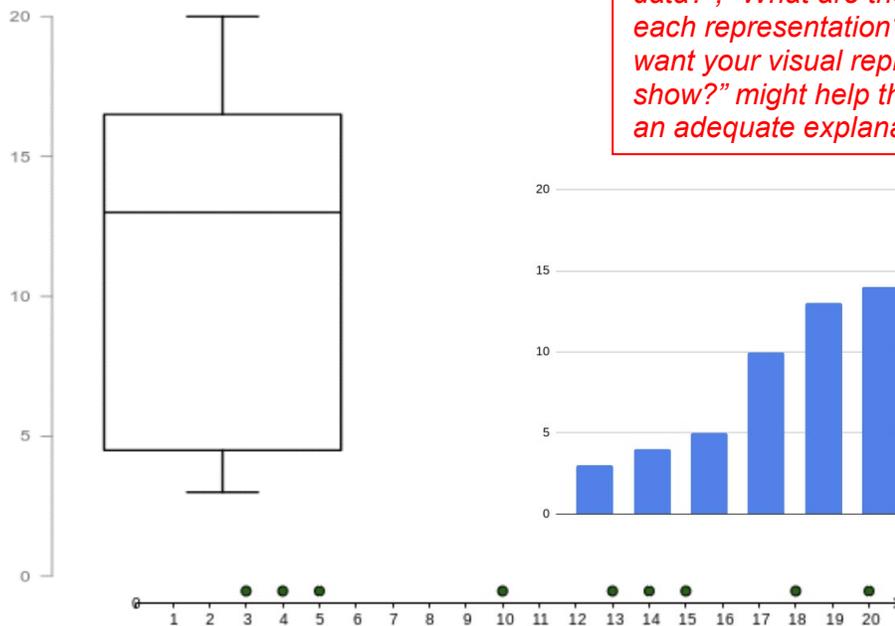
The student created a visual representation of their data, but the large outlier makes it hard to interpret the data. Asking questions such as "What does this visual representation tell us about the data?" "What is this visual representation not displaying?" "What do you want your visual representation to show?" would benefit the student.

Student C



The student question was, "How many times does it take you to perfect a skill you just learned?"

The student created four visual displays but did not explain why their visual representations are appropriate. Asking questions such as "What does this visual representation tell us about the data?", "What are the pros and cons of each representation?", or "What do you want your visual representation to show?" might help the student provide an adequate explanation.



Represent Mathematically Checklist

Requirement	Fulfilled?
I completed and compiled the data collection to be included in the report.	<input type="checkbox"/>
I described any modifications that may have been necessary in my data collection.	<input type="checkbox"/>
I generated the necessary statistics and interpreted each of them.	<input type="checkbox"/>
I chose a data display that is appropriate for the data set.	<input type="checkbox"/>
Teacher Consultation:	_____

In the **Analyze and Interpret Results** section, you will include an interpretation of your data. Use the measures you calculated above to support your response to the questions posed. The **Analyze and Interpret Results Checklist** can be used to ensure you have the necessary components before moving on to the remaining parts of the task.

Analyze and Interpret Results

ACTION 8



Choose the measure of central tendency and spread appropriate to your data. Explain your rationale.

[Students should state which measure of central tendency and measure of spread is most appropriate to describe their data (i.e., mean and standard deviation or median and IQR). Students should then provide a brief explanation based on mathematical evidence to support their claim. The following student responses seem to suggest that the teacher should emphasize that a measure of central tendency and spread is specifically required in Action 8. Student C did attempt to specify a measure of central tendency, but the response needs a measure of spread and further explanation to make it adequate. Students already calculated each of these measures, however it seems there is a gap in understanding of what they mean.]

Student A

My column chart, because it visualizes the information the best, making it easy for you to see all of the information, while showing it all accurately

Student B

The dot plot is more accurate in my opinion as it easily shows the widespread use of the different amount of books people have read in one year.

Student C

I would choose to use the mean and or the mean since I don't have a mode due to the not repeating numbers.

ACTION 9

Decide how you will determine and address outliers. Provide a summary of the evidence you considered when addressing outliers.

[Here, again, these students seem to lack an understanding of how an outlier can mathematically be determined from the data. It might be beneficial for teachers to remind students how to perform the calculations to determine if outliers exist.]

Student A

If the outliers are practicing more than the others, then I can assume that they are practicing the violin more than others, meaning that they are most likely better violinists. If the outliers are practicing less, then I can assume that they are slacking off, and are worse than the other violinists.

Encourage the student to explain how they will determine outliers from their data set.

Student B

I will address outliers as mostly a joke. For instance it is very hard to read 365 books in a year and therefore I don't consider it to be a serious answer. However an outlier of 18 is possible and I believe it to be a great outlier.

Encourage the student to explain how they will determine outliers from their data set. While 365 is far away from the rest of the data, is 18 truly a mathematical outlier?

Student C

I can address the outliers by picking out the most frequent times people take to perfect a skill then forget about the least ones. The most frequent terms determine how high your skill level is.

The teacher should encourage this student to mathematically determine outliers, explaining that even small or large numbers outside the "most frequent terms" can still fit inside the data set without being an outlier. By removing perceived outliers, we may unintentionally be biasing data and skewing results.

QUESTION 3



Recall in **Action 1** that you shared the purpose of the comparison you selected.

What story did the data tell relative to your purpose?

Who would benefit from these data, and how would they benefit?

Student A

What story did the data tell relative to your purpose?

Based on my data, I can see who is practicing the violin more in my class, and I can also assume based on this data that the people spending more time every week practicing the violin are better at playing the violin

Who would benefit from these data, and how would they benefit?

Our teacher would benefit from this data because it would show her who in our class needs to spend more time practicing than others.

The teacher might encourage the student to consider how they could quantify being "better at playing the violin." This is an interesting bi-variate statistical question to explore.

Student B

What story did the data tell relative to your purpose? *The story that my data tells me is that most people don't read many books.*

Who would benefit from these data, and how would they benefit? *Maybe schools, as they could use it to assign more literature causing the amount of books read to go up.*

The teacher might want to have the student consider who would benefit from reading more books and how they might the benefit? Is there any way to quantify this question?

Student C

What story did the data tell relative to your purpose? *The story told that everyone can take different times to get a skill clean and not everyone is going to be the same.*

Who would benefit from these data, and how would they benefit? *A cheerleader would benefit from this data because it's showing to never get discouraged when learning a new skill because it can take many tries and they aren't the only one.*

The teacher might want to have the student explore or explain further what it means to "get a skill clean." How could this be measured? What skill is being investigated? Are different skills more complex and take longer to master, if so, how would this affect the data collected?

Analyze and Interpret Results Checklist

Requirement	Fulfilled?
I selected the measures of central tendency and spread that are accurate and appropriate for the data.	<input type="checkbox"/>
I explored the possible effects of outliers using the shape, center, spread, and context of data sets.	<input type="checkbox"/>
I told the story of the data that accurately interprets and uses the results of the analysis to explain how this information can be useful.	<input type="checkbox"/>
I identified the audience who would benefit from this data.	<input type="checkbox"/>
Teacher Consultation:	_____

In the **Validate and Report** section, you will add on to the document you started above and clarify the voice and audience. In the **Validate** section, you will explain whether the data were sufficient for addressing your statistical question or how you would improve the process in the future. You will also create an appropriate way to report the findings from your work to the intended audience (e.g., through a short video, pamphlet, or other visual). The **Validate and Report Checklist** can be used to ensure you have the necessary components for completing the task.

Validate and Report

Action 10



Validate your model using these guiding questions.

- Did the data sufficiently address the question you were investigating? Why or why not?
- If you were to try to collect data to answer your statistical question again, how would you adjust your process, your question, and/or how you analyzed the data to provide better information to the audience interacting with your data?

Student A

- Did the data sufficiently address the question you were investigating? Why or why not? **It did because I can now see how many hours different people are spending practicing the violin**
- If you were to try to collect data to answer your statistical question again, how would you adjust your process, your question, and/or how you analyzed the data to provide better information to the audience interacting with your data? **I would ask people in other classes too, and then see which classes spend more time practicing the violin**

The student adequately answered the questions. However, the answers were short and did not show much reflection. The teacher might want to encourage the student to think about the second part of this question by asking question such as, "What could we do further with the analyzed data?" or "What might happen if other classes were asked?" or "Would there be any bias when asking other classes?" or "Are they at a different skill level?"

Student B

I don't think so as I was trying to find the personality and vocabulary usage of different people who read books and the varying amount of books they read. I would adjust it by asking what type of vocabulary they use.

The student seems to need some support in revising their statistical question. The teacher could support the student by helping the student narrow down their questioning into a measurable idea. The student started with a question, "How many books do people read in 1 year?" which doesn't relate sufficiently to address what they now claim they were trying to investigate.

ACTION 11



Communicate your results in a manner and format that are appropriate for the audience and purpose you identified in Questions 2 and 3, in a way that most clearly illustrates your findings.

[Teachers should be looking for responses that are supported with the mathematical evidence calculated and represented. Teachers can support students by revisiting Action 6 and reviewing the statistical analysis, ensuring that students were able to give contextual meaning to their calculations. A couple of typical (but inadequate) responses are shown below.]

Student A

I found that the people that generally practice the violin more after class tend to play better while they are in class

Student B

Although i believe reading is interesting my peers are not as interested and would rather spend time doing other things and therefore i cannot answer my question as people don't read unless it's necessary.

Validate and Report Checklist

Requirement	Fulfilled?
I provided a rationale showing how the data sufficiently addressed the question at hand or I articulated a clear understanding about how to improve the data collection and analysis in the future.	<input type="checkbox"/>
I clearly communicated the results in a way that is appropriate for the intended audience.	<input type="checkbox"/>
Teacher Consultation:	_____