

Annotated Examples: M151 Modeling with Geometry

The examples that follow are intended to illustrate how the learning principles are used to support students' engagement with the content and practices outlined in this badge. These examples do not provide comprehensive coverage of those expectations, but rather elevate some of the learning principles that are less likely to be part of published curricular materials for mathematics instruction. The examples that follow were developed by the Math Badging writing team, unless otherwise specified. These are a small sample of types of learning activities that can be done with students, both in and out of a traditional classroom setting.

Example 1: Eco-Friendly Product

Students are given a scenario like this:

Part 1: Design a Controller

You are a small business owner and you will create a new controller design for your favorite gaming system. You plan to sell your controller in an online store to reach a greater market.

- 1. Sketch a design of the controller. Consider the shapes you'll use in your redesign: a prism, cone, pyramid, or some combination of these shapes (or any three-dimensional geometric figure you can describe).*
- 2. Determine the dimensions. How much material will this design need?*
- 3. Explore how companies are approaching eco-friendly products. What did you learn? Why might this be important?*
- 4. What do you want to aspire to as you reflect on the design of your product?*

Invite students to revisit and redesign their item.

Part 2: Design a Package

Your job is to design an eco-friendly package for shipping the controller you designed.

- 1. Explore how other products are currently packaged. How eco-friendly are the current designs? How can the current design be improved upon? What elements in your analysis would make the packaging eco-friendlier?*
- 2. Design an eco-friendly way to package the controller you imagined in Part 1. Make note of the packaging shape and dimensions.*
- 3. Prepare a presentation that illustrates the design of the controller and packaging.*
Within your presentation:
 - a. prove the controller fits.*
 - b. share how your design is eco-friendlier than what is currently used.*
 - c. share the benefits and drawbacks of packaging items like your design compared to the ones we see in stores.*

Sample Learning Activity

Invite students to share hobbies and interests they have outside of school. In this task students have the opportunity to explore some of the decisions a small business owner and designer has to make. Part 1 offers an opportunity for students to explore and design a new controller. They may use a collection of materials, card stock, pre-designed cardboard cylinders, and paper as well as computer-aided design software to develop their controller design. Give students opportunities to research and brainstorm product designs, with at least one cycle of feedback.

Here are some resources for students to explore:

[Ecofriendly Shipping Supplies](#)

[What Does Eco-Friendly Actually Mean?](#)

[What Does Eco-Friendly Mean?](#)

[Xbox One's Production Cost Revealed](#)

[24 Eco Friendly Packaging Examples that Benefit your Brand](#)

[18 Creative Sustainable Packaging Design Examples](#)

Here are some questions to facilitate a feedback discussion:

- *What do you appreciate about this approach?*
- *What questions does this approach surface for you?*
- *Do you think this design is reasonable? Why or why not?*
- *What can you take from this approach to apply to your own work?*

If not raised by students, consider inviting them to think about the ramifications of mass producing a design. Give students an opportunity to redesign their product based on insights gained from feedback sessions.

Part 2 of this task offers students an opportunity to explore current packaging practices and reflect on eco-friendly alternatives.

Offer them the opportunity to work in pairs or small groups to complete the task. Give the option of using chart paper, Google Slides, or other creative outlets to showcase their work. Give students the opportunity to return to their work to revise, as necessary.

In this example, students are:

- engaging with cognitively demanding tasks in heterogeneous settings. Students engage in iterative cycles using geometric concepts and methods to design a product and its packaging (LP 1).
- set up to have social interactions, as they share and discuss their work with each other. These interactions may happen live in a physical classroom or asynchronously in a remote setting (LP 2).
- building understanding through application of geometric concepts of surface area and volume of two- and three-dimensional aspects (LP 3).

- able to explore areas of interests and develop a lens for aspirations, as they explore the world of product design and packaging (LP 4).
- connecting relevant learning to authentic situations (LP 6).
- able to use technology as a tool, as they employ computer-aided design software to perform the design of their product and packaging (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Logical thinking:** [Language and Evidence](#). (OT.Crit.2a)

Example 2: Roses for Rosa

Rosa and her family are planning for her Quinceañera. Each of the 15 tables at Rosa's reception will have a centerpiece—a cylindrical glass vase filled one-third of the way with glass marbles holding a rose in place.

- The glass vases are 10 inches tall with a 4 inch diameter.
 - Rosa can choose between marbles that are 12mm or 16mm in diameter.
 - The 12mm marbles cost \$11.99 for 340 pieces.
 - The 16mm marbles cost \$11.99 for 140 pieces.
 - Rosa spent \$110 on the vases.
 - The florist is not charging Rosa for the 15 roses for the centerpieces.
1. If Rosa chooses 12mm marbles, how many bags will she need? How many bags will she need if she chooses 16mm marbles?
 2. Rosa's budget for the centerpieces is \$250. Will she be able to stay within her budget?
 3. The marble company also sells 8mm marbles (1100 pieces for \$11.99). Would you recommend Rosa buy these marbles instead?

Here are some questions to facilitate a feedback discussion:

- *What are we really trying to find out in this problem?*
- *What do you need to know to begin solving the task?*
- *Besides the information given in the problem, what else might you need to know?*
- *Did other questions surface during the process of finding a solution?*
- *What can you take from solving this task to apply to your own life?*

To enhance student thinking, have student consider:

- *What would change if the prices of the bags of marbles were not the same?*
- *What reasonable prices would make the 12 mm the better price? The 16mm?*
- *How much water would fill the remaining space in each vase option?*
-

Worked Student Example

1. If Rosa chooses 12mm marbles, how many bags will she need? How many bags will she need if she chooses 16mm marbles?

$10\text{cm} = 100\text{mm}$
 $25\text{cm} = 250\text{mm}$
 $r = 5\text{cm} = 50\text{mm}$
 $V = \pi r^2 h = \pi (50)^2 \cdot 250$
 $V = 785,000\text{mm}^3$
 $V = 208,333 \frac{1}{3}\text{mm}^3$

12mm
 $V = \frac{4}{3}\pi r^3$
 $V = 288\pi\text{mm}^3$
 do I need to worry about the space between?

$V = 723 \frac{4}{108}$ marbles needed - for 12mm = 15cps
 Rosa needs ~32 packs of 12mm marbles.
 Rosa needs ~33 packs of 16mm marbles

16mm
 $V = \frac{4}{3}\pi r^3$
 $V = 682 \frac{2}{3}\pi\text{mm}^3$
 $\frac{208,333 \frac{1}{3}\text{mm}^3}{682 \frac{2}{3}\pi\text{mm}^3}$
 $= 305 \frac{3}{16} \cdot 15$
 $= 4577 \frac{163}{1276}$
 $\underline{140}$

In this example, students are:

- engaging with a cognitively demanding task. It offers little in the way of traditional diagrams or measurements that support routine computations. Instead, students must visualize or create their own diagram, including making assumptions or researching information about the space marbles can fill (LP 1).
- set up to have social interactions, as they share and discuss their work with each other. These interactions may happen live in a physical classroom or asynchronously in a remote setting (LP 2).
- building conceptual understanding through reasoning. Students must construct a series of logical arguments and assumptions in their modeling as they use circles and volumes of cylinders to model this situation (LP 3).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:

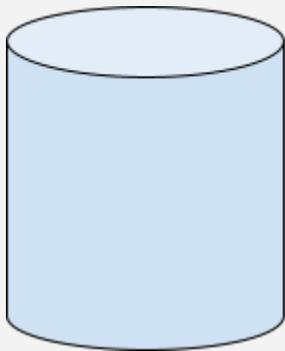
-  **Logical thinking:** [Language and Reasoning](#) (OT.Crit.2)

Example 3: Similar Solids

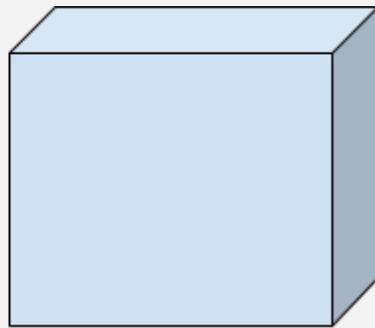
Students are given a problem like this:

What are some ways these two physical representations of solids are alike? different?

A



B



Now consider the following questions.

1. *What information would you need to calculate the volume of each solid?*
2. *What is the same and different about how you would find the volume of each solid?*

These images show the same number of coins arranged in different ways:

C



D



3. *How are the two coin stacks different from each other?*
4. *Does either stack resemble a geometric solid? If so, which stack and solid?*
5. *How do the heights of the two stacks compare?*

6. *How do the volumes of the two stacks compare? Explain your reasoning.*

(Adapted from [Cylinder Volumes](#) and [Cross Sections and Volume](#) by Illustrative Mathematics, 2019)

Sample Learning Activity

Students are asked to look at the solids depicted in A and B, and to answer the questions that follow independently. Students then collaborate with a partner while a teacher observes and selects student responses to share with the whole class for discussion, such as these for Question 2:

- “The figures are the same because they are both 3-dimensional.”
- “The figures are the same because they both have a height.”
- “The figures are the same because they will both have the same volume.”
- “The figures are the same because you can calculate the volume the same way for both.”
- “The figures are the same because you can find the volume by multiplying the area of the base times the height.”

As these responses are selected and shared, the teacher may pose discussion questions like these:

- “Do you agree with this statement? Why or why not?”
- “What adjustments might you make to the statement to make it more accurate?”

Students are then given time to examine figures C and D and respond to the questions independently before conferring with a partner. Time could then be focused on Question 4; the teacher could ask students to choose whether they think the volumes are the same or different, and facilitate a debate where students offer logical arguments one way or the other to convince their classmates.

In this example, students are:

- given a non-routine task. Instead of being given figures or models labeled with specific dimensions (e.g., “the height of the cylinder is 30 cm”), students must make assumptions and think conceptually about the meaning of volume (LP 1).
- building a conceptual understanding through reasoning. Students’ conceptual understanding is strengthened by comparing figures. Instead of computing, they must use the definition of volume to think about missing information, form comparisons, and explain their thinking (LP 2).
- engaging in social activities as they share their reasoning with a partner, discuss each other’s ideas as a class, and debate a claim about volume (LP 3).

Through the implementation of this learning activity, look for opportunities to support students’ growth and development on this XQ competency:



- **Mathematical Modeling:** [Understanding Models](#) (FL.MST.2)

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