

M104: Modeling with Functions of Exponential Type

Pre-Badge Task

Teacher Scoring Guide

New Badge Alert!

This task will provide a preview of what we will be learning during the M104 - Modeling with Functions of Exponential Type Badge. You already know so much math, so use this task to share any understanding that you have before we start our learning. You will have a chance to do the same task at the end of the badge to see how your thinking has evolved.

The Task

You decide to purchase your first car. You decide on a Subaru Crosstrek, which is valued at \$21,500 at the time of purchase. The table below indicates the value of the car, in dollars, over the first 4 years that you own it.

Year	0	1	2	3
Value (dollars)	21,500	18,920	16,650	14,652

The following pages include sample student responses in **blue font**, as well as additional scoring comments in **red font**.

QUESTION 1 What do you notice about the data provided in the table above?



Answers will vary. Possible student responses:

- I notice the value of the car decreases each year
- The amount of decrease in value is bigger in the first year than in the last year
- The first year had a decrease in value of \$2580
- The second year had a decrease in value of \$2270
- The third year had a decrease in value of \$1998
- The car depreciated a total of \$6848 in three years

QUESTION 2 What is the approximate percentage that the value of the car is depreciating each year?



*While this question didn't ask students to show their work, the teacher might prompt students to do so. Of concern is that a student thinks about the total amount of depreciation across the three years (\$6848) and divides that amount by the beginning value (\$21,500) to come up with a rate of about 32%. Then they divide this percent by 3 to get a rate of about 10.6%. This doesn't adequately address the question in terms of the **annual rate of depreciation**.*

Possible student response:

The approximate percentage that the value of the car is depreciating each year is 12%. From year 0 to 1, the depreciation was \$2580. Dividing 2580 by 21,500 is 12%. From year 1 to 2, the depreciation was \$2270. Dividing 2270 by 18,920 is ~ 12%. From year 2 to 3, the depreciation was \$1998. Dividing 1998 by 16,650 is 12%.

QUESTION 3 Write an equation that represents the value, y , (in dollars) of the car after x years.



Possible student response:

$V = P(1 - r)^t$ where V is the current value, P is the original price, r is the annual rate of depreciation and t is the number of years since the car was purchased.

$$\text{So, } V = 21500(1 - 0.12)^t$$

$$\text{or } V = 21500(0.88)^t$$

QUESTION 4 Graph the curve represented by your equation. You can use graph paper or technology to create your graph.



[Attach a file with your work or describe how to find it if it is stored online.]

Possible student response:



QUESTION 5 Why is the graph shaped this way? Explain your answer.



Possible student responses:

Because the depreciation formula is an exponential function. The graph decreases quickly at first and then slows down, eventually approaching zero.

OR

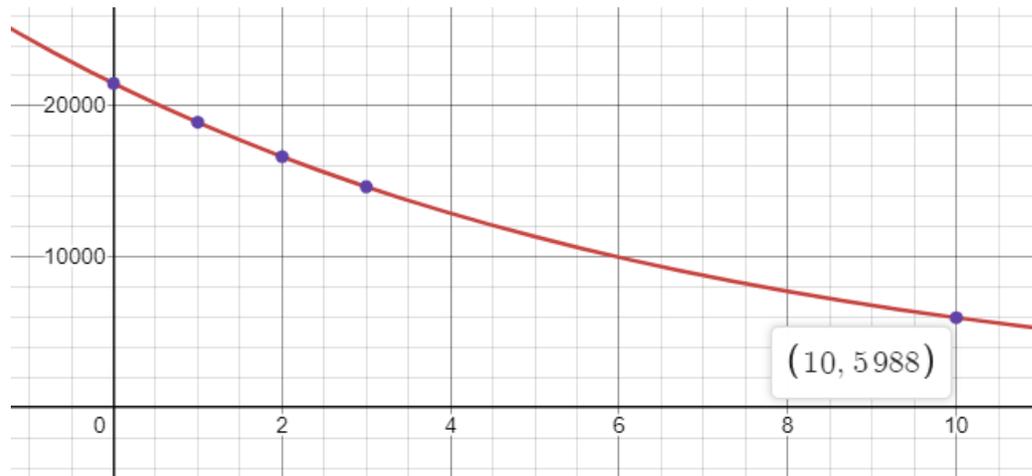
The graph is curved because the difference between each point is not the same, which is what it would be if it were linear.

QUESTION 6 How many years will it take for your car to be worth less than \$6000? Explain how you determined this.



Possible student responses:

I extended the graph, using Desmos, until I saw the value dip below \$6000. It appears that it will take about 10 years for that to happen. The value is about \$5988 at 10 years.



OR

I plugged 6000 into the equation for V and solved for t to find the number of years it would take the car to be valued at \$6000.

$$6000 = 21500(0.88)^t$$

$$0.27907 = 0.88^t$$

$$\log 0.27907 = t \log 0.88$$

$$\frac{\log 0.27907}{\log 0.88} = t$$

$9.98 = t$, so it will take about 10 years

QUESTION 7 Does the car lose the same dollar amount each year? Explain your answer.



Possible student response:

No. The dollar amount lost decreased each year, even if the percent depreciation was the same.

From year 0 to 1, the car lost \$2580 in value.

From year 1 to 2, the car lost \$2270 in value.

From year 2 to 3, the car lost \$1998 in value.