

M202: Rational Exponents and Complex Numbers Post-Badge Task

Reflecting on Badge Learning

Now that you have learned more about rational exponents and complex numbers, respond to the questions that follow. These are the same questions you responded to before the start of this badge.

The Task

QUESTION 1 Consider the following powers of 4:

$$4^{-1} = \frac{1}{4} \quad 4^0 = 1 \quad 4^1 = 4 \quad 4^2 = 16$$

What do you think $4^{\frac{1}{2}}$ would be? Explain your reasoning.



Answers may vary. Possible student response:

I think $4^{\frac{1}{2}} = 2$ because if you square 2, you would get 4.

QUESTION 2 Abigail and Ben are comparing their methods to solving for the roots of

$$f(x) = x^2 + 4x + 13.$$

Abigail's work:

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 52}}{2}$$

Ben's work:

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(13)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 52}}{2}$$

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$$x = \frac{-4 \pm \sqrt{-36}}{2}$$

$$x = \frac{-4 \pm 6i}{2}$$

$$x = -2 \pm 3i$$

$$x = \frac{-4 \pm \sqrt{36}}{2}$$

$$x = \frac{-4 \pm 6}{2}$$

$$x = 1 \text{ and } x = -5$$

After reviewing Abigail and Ben's work, answer the following questions:

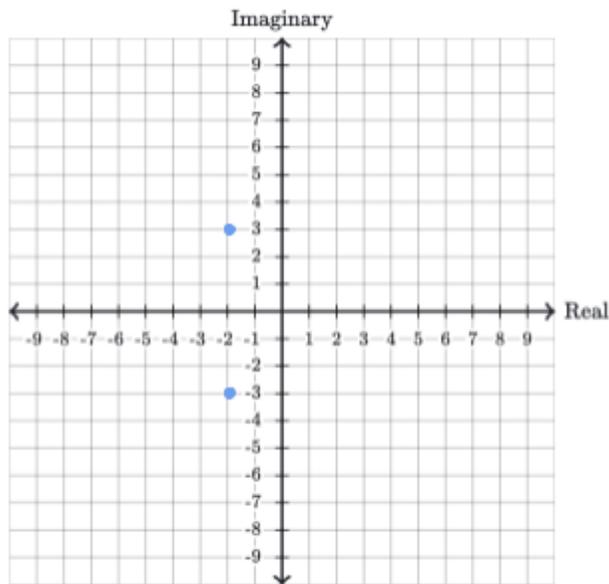
- Who is correct? Explain.
- Graph the two correct solutions on a complex plane.



Who is correct and why:

Abigail is correct. Ben made a mistake in the third line when he subtracted 16 and 52, which should give him -36 instead of 36. This error did not give him a complex solution, as it should have.

Graph:



QUESTION 3 A researcher is studying bacterial growth and finds that the number of bacteria in a sample follows the function: $P(t) = 100(2^{\frac{t}{3}})$ where t represents time in hours.

- Solve for $P(900)$. Interpret this solution in the context of bacteria growth.
- Suppose another researcher uses a different model: $Q(t) = 100(4^{\frac{t}{6}})$. Compare the growth rates of P and Q . Which model predicts faster growth? Why?

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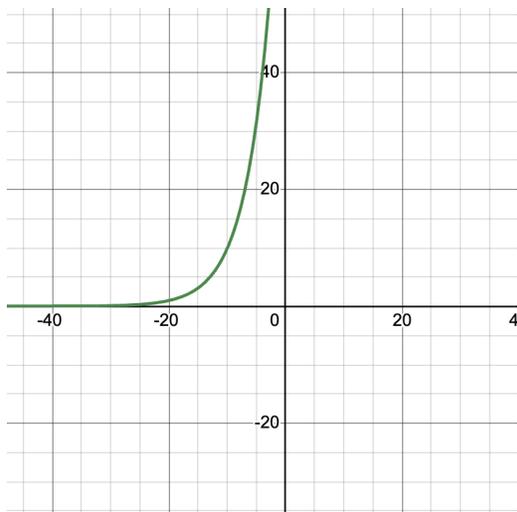
$$P(900) = 100(2^{\frac{900}{3}}) = 100(2^{300}) = 2.03703598 \times 10^{92}$$

After 900 hours, the number of bacteria will have grown to $2.03703598 \times 10^{92}$.

Answers may vary. Possible student response:

If we compare the graphs of P and Q , the two graphs align perfectly with each other,

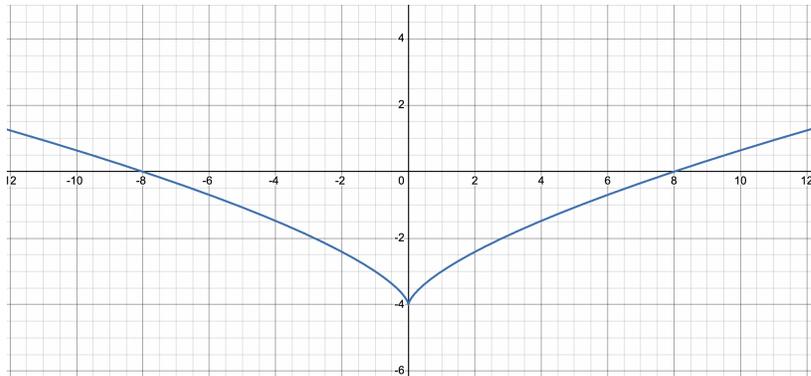
meaning they are the same function. This is because $4^{\frac{1}{6}} = (2^2)^{\frac{1}{6}} = 2^{\frac{2}{6}} = 2^{\frac{1}{3}}$.



QUESTION 4 Consider the function $h(x) = x^{\frac{2}{3}} - 4$.

- Graph $h(x)$ by hand or using technology.
- Identify any intercepts and describe the overall shape of the graph.
- How does the graph of this function differ from the graphs of $f(x) = x^2$ or $f(x) = x^3$? What patterns do you notice?

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Answers may vary. Possible student response:

There are two x-intercepts at -8 and 8. There is one y-intercept at -4. The graph is symmetric about the y-axis, showing two smooth curves on each side.

This graph is similar to $f(x) = x^2$ in that the end behavior is also positive, positive. The graph is similar to $f(x) = x^3$ in that the shape of each side looks similar to its two smooth curves.

QUESTION 5 Consider $A = 4 - i$ and $B = 3 + 2i$. Compute the following:

- $A + B$
- $B - A$
- A^2
- $(A - 4)^2$

How do your solutions for A^2 and $(A - 4)^2$ compare? What do you notice?



$$A + B = (4 - i) + (3 + 2i) = 7 + i$$

$$B - A = (3 + 2i) - (4 - i) = -1 + 3i$$

$$A^2 = (4 - i)^2 = 16 - 8i + i^2 = 16 - 8i - 1 = 15 - 8i$$

$$(A - 4)^2 = (4 - i - 4)^2 = (-i)^2 = -1$$

Answers may vary. Possible student response:

The solution for A^2 contains an imaginary number value while $(A - 4)^2$ does not. By subtracting 4 in $(A - 4)^2$, it leaves us to square $-i$, resulting in an integer value solution.

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QUESTION 6 What else did you learn that was not reflected in Questions 1-5?



Answers will vary. Possible student responses:

- I can rewrite expressions involving radicals and rational exponents using the properties of exponents.
- I can solve equations using properties of exponents and radicals.

QUESTION 7 Compare your answers to these questions now to your answers from the earlier attempt. What do you notice about your progress?



Answers will vary. Look for students to explain what they know now compared to what they knew when responding to the pre-test.