

Annotated Examples: M205 Trigonometric Functions

The examples that follow are intended to illustrate how the learning principles are used to support students' engagement with the content and practices outlined in this badge. These examples do not provide comprehensive coverage of those expectations, but rather elevate some of the learning principles that are less likely to be part of published curricular materials for mathematics instruction. The examples that follow were developed by the Math Badging writing team, unless otherwise specified. These are a small sample of types of learning activities that can be done with students, both in and out of a traditional classroom setting.

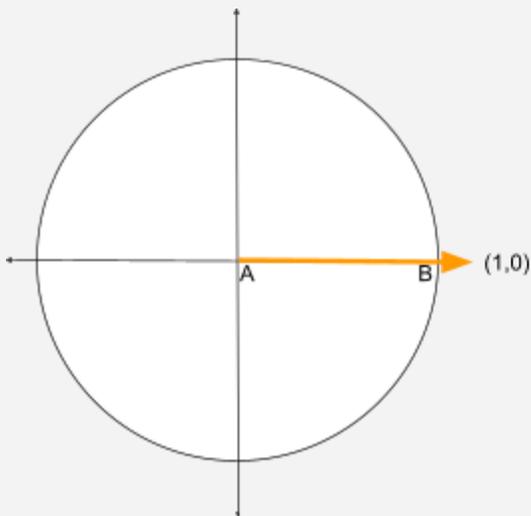
Example 1: The Unit Circle

Students are presented with a problem like this:

Part 1

Draw a circle on a coordinate plane centered at $(0,0)$ with a radius of 1. This is called a unit circle.

What is the circumference of the circle?



What is the length of the arc from point $(1,0)$ to the point $(-1,0)$? How do you know?

Part 2

Use the following steps to find additional arc lengths, angles, and coordinates on the unit circle.

- (1) Find the arc length from point $(1,0)$ to a different point on the unit circle.
- (2) Mark and label the point J.
- (3) Record the arc length in the table.
- (4) Draw ray AJ.
- (5) Determine the measure of the angle formed by ray AB and ray AJ. Record this in the table.
- (6) What are the coordinates of point J? Record the coordinates in the table.
- (7) Repeat steps 1-5 for 3 more points. Label them K, L and M.

Point	Arc Length from $(1,0)$	Angle Measure	Coordinates
J			
K			
L			
M			

Part 3

Definition of 1 radian: An angle of 1 radian is defined to be the angle, in the counterclockwise direction, at the center of a unit circle which spans an arc of length 1.

Given the definition of 1 radian, approximately where would 1 radian be on your circle? Mark a point there and label it point P.

If you haven't already, explore the relationship between the sine and cosine of the angle measures from your table and the coordinates of the associated points. What do you notice? How can you explain the relationship that exists, if any?

Sample Learning Activity

The purpose of this task is to explicitly connect arc lengths on the unit circle to the radian as a unit of measure.

Begin by giving students quiet time to make sense of the information given and process Part 1 of the task they have to complete. Provide them with physical tools, such as graph paper, rulers, or digital geometry software to experiment with relationships made visible by the unit circle.

After some individual time, encourage students to discuss progress with a partner or small group. Encourage students to make their thinking visible. Initially, focus a conversation on how students interpreted and used the given information. Consider pulling the class together to discuss and share students' labels and use of tools to build shared understanding of the known information about the triangles. Monitor and select student work that aids in the conversation.

Ensure that students' understanding of Part 1 is secured before having them explore Part 2. Monitor for, select, and share student work that embodies different approaches and levels of formality for discussion. Listen for the various ways that students determine how to complete different columns in their table in Part 2. Did they use trigonometry or algebra knowledge to determine the coordinates? Did they estimate the angles and arc lengths or are they precisely determined?

As students share their understanding and reasoning with classmates, invite them to make sense of each other's work. Consider the following questions:

- *What are some things you appreciate about this explanation?*
- *What questions does this explanation surface for you?*
- *What can you take from this explanation to apply to your own work?*

Following the sharing and discussion, give students an opportunity to revise their work and share with each other again. Focus this second discussion on changes or adjustments that they have made based on their peers' work.

In this example, students are:

- given a cognitively demanding task. It requires students to reason using the structure of the unit circle to justify claims about arc lengths, angle measures, and coordinates on the unit circle. The learning activity is structured to give all students access by inviting them to write explanations with varying degrees of formality and giving an opportunity for revision after engaging with some of their peers' explanations (LP 1).
- working collaboratively through structured opportunities to ask questions to each other to better understand the thinking process (LP 2).
- building conceptual understanding through reasoning, as they must relate the unit circle and the work they did in right triangles in trigonometry (LP 3).
- encouraged to have agency, as they choose their solution pathways and are given space to reflect on how their approach might change based on engagement with their peers' work (LP 4).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



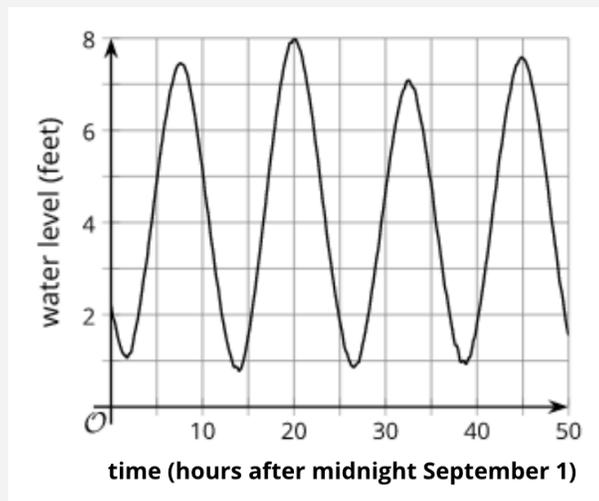
- **Computational Thinking:** [Analyzing patterns](#) (FL.MST.1e)

Example 2: Sine and Cosine

(Adapted from IM Alg 2 6.19 Summary)

Students are presented with a scenario like this:

Have you ever gone to the beach on a summer day and seen a yellow flag on the shore? This flag is an indication of moderate surf, signaling that the tides may be high and swimmers should be cautious when entering the water. Oceanographers are people who research and study the different conditions of the ocean. One important job of oceanographers is to study the waves, currents, and tides. These reports on ocean activity help us to know when the ocean is safe and allowed to be open to the public for activities such as fishing, boating, and swimming.



1. Above is an image of the water level in Bridgeport, Connecticut, over a 50 hour period in September. Using what you know about trigonometric functions, you will make sense of this information and do the job of an oceanographer for the day!
 - a. What initial observations do you have about the water levels shown by the graph?
 - b. Based on the image, what is the period for this graph? Explain how you know.
 - c. Use Desmos to create an equation to model the relationship between the water level in feet and the number of hours after midnight on September 1st. What equation did you come up with? Explain how you created and refined your equation to get a close fit to the graph shown in this problem.
 - d. How might swimmers, boaters, or fishers use the information in your graph to make informed decisions about their activities? Is there anybody else who might be interested in the information shown by the graph? If so, who?

Sample Learning Activity

Introduce the lesson by displaying the given image. Ask students: “*What do you notice? What do you wonder?*” Chart student responses close to the image, in a place that is visible to the whole class. Encourage them to elaborate in order to make the ways they are attending to structure visible to themselves and their classmates. Invite them to record some of these responses and elaborate as they examine question 1a.

Give students time to engage in the remaining questions individually, followed by partner or small group work. Monitor and select varied approaches to the prompts to share for whole class discussion.

As students share their approaches with classmates, invite them to make sense of each other’s work, particularly when engaging with how they approached Question 1c. Consider the following questions:

- *What are some things you appreciate about this approach?*
- *What questions does this approach surface for you?*
- *What can you take from this explanation to apply to your own work?*

Following the sharing and discussion, give students an opportunity to revise their work or build upon their work to share with each other again.

In this example, students are:

- given a cognitively demanding task. They are asked to consider how to create a model of data that is periodic and use that information to make predictions (LP 1).
- working collaboratively in pairs and sharing their findings with the class. Students are encouraged to ask questions of each other to better understand the thinking process, as well as given space to learn from their peers (LP 2).
- engaging in learning activities that emphasize reasoning and explanation over performing computations with trigonometric ratios (LP 2).
- asked to explain the meaning of parameters for a trigonometric function that models a real-world situation (LP 3).
- invited to consider how the data related to tides can be used to make decisions about safety with different ocean activities (LP 6).
- employing a variety of tools, such as calculators and geometry software, to analyze and interpret trigonometric functions (LP 7).

Through the implementation of this learning activity, look for opportunities to support students’ growth and development on this XQ competency:



- **Making Meaning:** [Synthesizing information](#) (FL.ID.1.a)

References

Illustrative Mathematics. “Lesson 19 Beyond Circles.” Illustrative Mathematics, Kendall Hunt, 2019, <https://im.kendallhunt.com/HS/teachers/3/6/19/index.html>.