

# Annotated Examples: M103 Modeling with Functions of Quadratic Type

The examples that follow are intended to illustrate how the learning principles are used to support students' engagement with the content and practices outlined in this badge. These examples do not provide comprehensive coverage of those expectations, but rather elevate some of the learning principles that are less likely to be part of published curricular materials for mathematics instruction. The examples that follow were developed by the Math Badging writing team, unless otherwise specified. These are a small sample of types of learning activities that can be done with students, both in and out of a traditional classroom setting.

## Example 1: Ball Throwing

Students are given a problem like this to work on in groups:

*Work with your group to develop a model that describes the height of a ball thrown at a given time during the throw. Have one person in the group throw a ball while another records a video of the event. Make sure that the video includes the entire path of the ball from the throw to when it hits the ground. Prepare your findings to share with the class.*

## Sample Learning Activity

Students could be given the largely unstructured task above, along with the tools needed to carry it out: a ball, recording technology, and tools to make mathematical models, like spreadsheets and access to graphing software. Consider bringing the class together at strategic points to facilitate progress toward the goal of developing a mathematical model.

In these instances, monitor, select, and sequence groups to report on their efforts.

The following questions can be used to facilitate discussion early in the process:

1. Watch a selected group's video. Ask students to describe what they notice while watching the video. Listen for descriptions of key elements, such as the height of the ball upon release, the highest point the ball reached, and when and where the ball landed.
2. Ask students, "*How might we represent this mathematically?*"  
As students share, make note of ideas that surface. When they use hand signals or share descriptions that allude to the graph of a quadratic function, encourage students to sketch what that might look like.
3. Once a sketch of a graph is offered, ask students, "*What quantities are captured here?*" in order to develop insights into what quantities they will work on to define and then gather data.

4. Use the list of what students noticed in number 1 above to frame the next phase of investigation.  
For example, you may build from a statement like “The ball hits a high point before coming back down” to ask, “*How high was the ball above the ground at its highest point?*” Invite students to discuss and brainstorm the following question: “*What makes finding this difficult and how can you persevere?*”
5. Ask students to reflect on the question “*What can you take from this discussion to apply to your own work?*” before continuing with their investigation.

When ready, give time for groups to share their models with the class. Here are some questions to facilitate this discussion:

- *What do you appreciate about this approach?*
- *What questions does this approach surface for you?*
- *Do you think the model is reasonable? Why or why not?*
- *What can you take from this approach to apply to your own work?*

Following the discussion, students are given an opportunity to reflect on and revise their work.

In this example, students are:

- given a cognitively demanding task. It requires students to model projectile motion using a quadratic function. They must make many assumptions to design, investigate, and define their own variables. The exercise begins with a broadly accessible task and prompts—“Ask students to describe what they notice”—that support learners with varied prior learning activities (LP 1).
- set up to have social interactions, as they work in small groups and discuss their work with each other. These interactions may happen live in a physical classroom or virtually (LP 2).
- deepening their understanding of functions used to model data. Students also have the opportunity to make connections between key features of the context, graph, and equation. As they consider the relationship between the quantities, students may recognize that a linear model has limited usefulness and therefore might prefer a nonlinear model, quadratic in nature (LP 3).
- able to explore skills and dispositions around what it means to model with mathematics. As they engage in this work, students have the opportunity to develop identities and interests in using mathematics to make sense of the world (LP 4).
- using technology as a tool, as they employ spreadsheet and/or graphing software to perform their analysis (LP 7).

Through the implementation of this learning activity, look for opportunities to support students’ growth and development on this XQ competency:



- **Interpreting Information:** [Extracting Ideas](#) (OT.Crit.1.a)

## Example 2: 3, 2, 1 Liftoff!

Students are given a problem like this to work on in groups:

### Part 1: Making Sense of Rocket Launches

1. Watch this video of [Grasshopper 744m Test| Single Camera \(Hexacopter\)](#).
2. Reflect: What questions does this clip bring to mind?
3. Sketch a diagram that illustrates the rocket's flight. Use the diagram you developed to sketch a graph.
4. Read the article "[Why SpaceX wants to land a rocket on a platform in the ocean](#)" by Joseph Stromberg.
5. What questions do the article and the video answer? What new questions does this surface for you?

### Part 2: Modeling with Quadratics

After some trials, SpaceX learned to control the flight of reusable rockets used to launch spaceships into space.

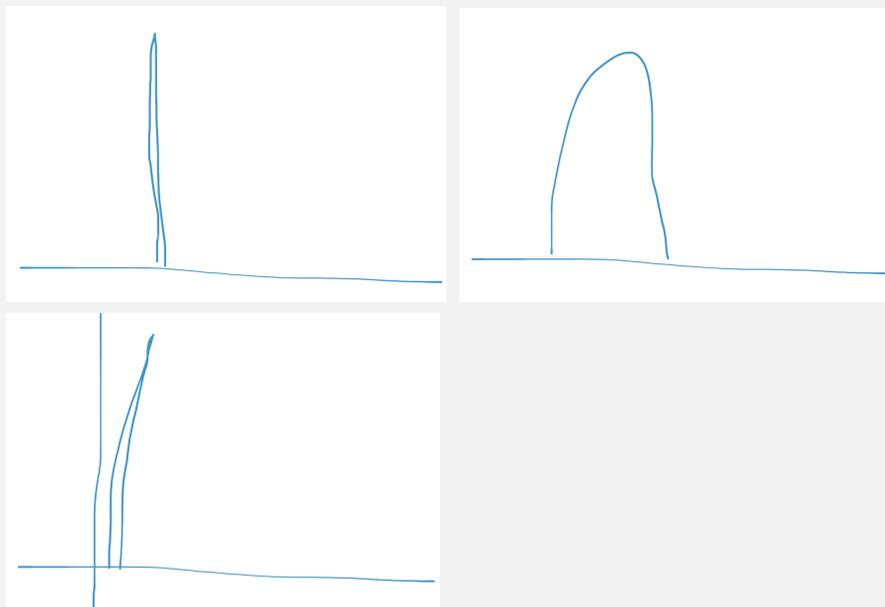
1. Watch and gather data from [SpaceX' April 2022 launch of Ax-1](#) [watch from about 3:26 to 3:36] to create a model for the flight of the Phase 1 rocket used in that mission.
2. Prepare to share your findings.
3. What new questions surface for you?

### Sample Learning Activity

#### Part 1: Making Sense of Rocket Launches

Begin by inviting students to share what they know about space exploration. Listen for stories where students express curiosity about launching objects into space: how are things launched, do these objects ever come back, and what happens to objects once they are launched into space? Use these themes to set up SpaceX's work with reusable rockets. Share the video of Grasshopper's 744m Test (1:36) from their early efforts in the development of these rockets.

Monitor students as they work to sketch the rocket's flight. Select student responses to highlight. Some possible approaches may take the following forms:



As students share their sketches with classmates, invite them to make sense of each other's sketches. Consider the following questions:

- *What are some things you appreciate about this sketch?*
- *What questions does this sketch surface for you?*
- *Do you think the sketch is reasonable? Why or why not?*
- *What do your horizontal and vertical axes represent?*
- *What can you take from this approach to apply to your own work?*
- *What are some quantities we could track and work to represent in mathematical ways? What questions could we answer with that information?*

As you bring the discussion of Part 1 to a close, give students time to make note of new insights they have gained into rocket flights. Consider amplifying ideas around what quantities they want to track and ask how they might measure or get that information.

## **Part 2: Modeling with Quadratics**

Share with students, "Since SpaceX's efforts in 2013 with the Grasshopper rocket, they have made tremendous progress."

Use thoughts students shared in discussion of Part 1 to set the stage for viewing the video of the April 8, 2022, Ax-1 Mission. This video tracks the time, speed (km/hr.), and altitude for both the Phase 1 and Phase 2 rockets. Encourage students to focus on the Phase 1 rocket.

Give students a few minutes of individual work time before encouraging them to work in a small group. Students should have access to spreadsheets and graphing software or calculators. As they join their small groups, ask students to take time to share and make sense of each other's approach. As students share progress made within their small groups, select a couple of strategies to share with the whole class. The focus of this whole group discussion is to provide space for students to

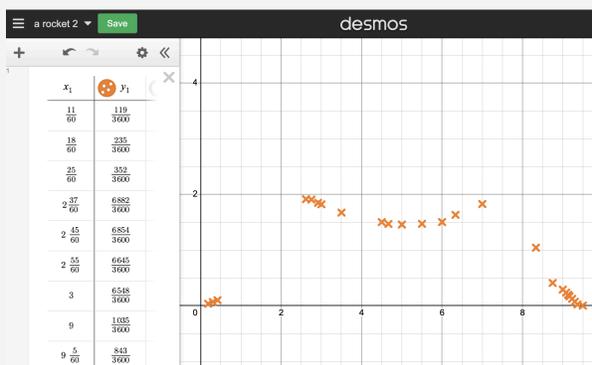
make sense of various approaches and gain insights into their own thinking. The goal is not to discuss a model just yet.

Consider using the following prompts to facilitate the conversation:

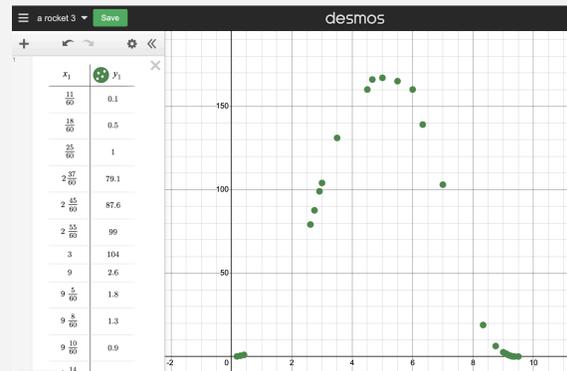
- *What makes sense about this approach?*
- *What questions does this approach surface for you?*
- *Do you think the approach (or model) is reasonable? Why or why not?*
- *What can you take from this approach to apply to your own work?*

Some possible graphs students may develop:

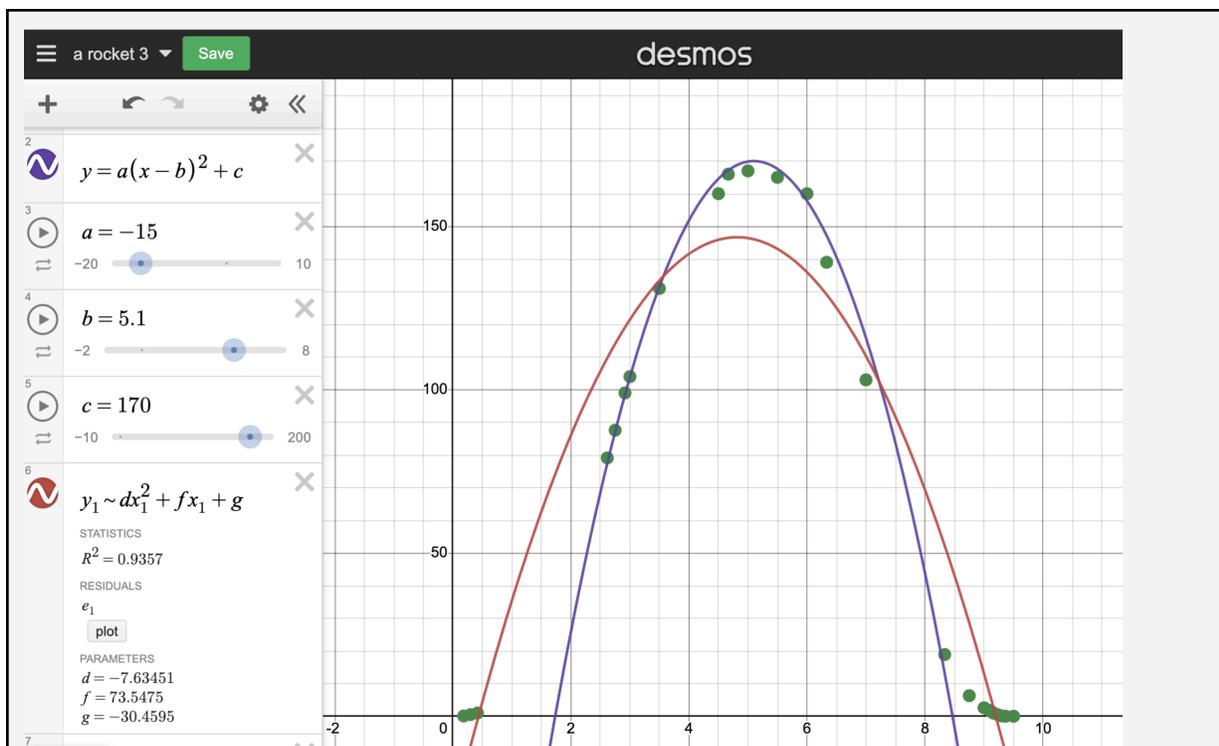
Speed (km/s) vs. time



Altitude (m) vs. time



Give students additional time to use insights gained from sharing to revise and further develop their models. The image below illustrates a manual fit (purple) quadratic and one from a regression analysis. As student groups work, monitor and note different approaches they take to find a quadratic model they feel best fits the data gathered. Select varied approaches to share with the whole class.



Consider using the following prompts to facilitate the conversation:

- *What's promising about this approach?*
- *What questions does this approach surface for you?*
- *Do you think the model is reasonable? Why or why not?*
- *What can you take from this approach to apply to your own work?*

Amplify aspects of the conversation that:

- attempt to explain the quadratic nature of this relationship.
- contextualize the key features of the graph.
- lead to the equation of this quadratic function.

Allow students the opportunity to return to their work to revise, as necessary.

Have students work in pairs to complete the task. Allow them the option of using chart paper, Google Slides, or other creative outlets to showcase their work.

In this example, students are:

- given a cognitively demanding task. It requires students to make sense of rocket trajectories and recognize which quantities can be modeled with quadratic functions. The task offers entry for students with a variety of prior learning activities by beginning with student-posed questions (e.g., “What questions does this clip bring to mind?”) and informal sketching of rocket trajectories (LP 1).

- set up to have social interactions, as they work in small groups and discuss their work with each other. These interactions may happen live in a physical classroom or virtually (LP 2).
- deepening their understanding of functions used to model data. Students also have the opportunity to make connections between key features of the context, graph, and equation (LP 3).
- able to explore skills and dispositions around what it means to model with mathematics. As they engage in this work, students have the opportunity to develop identities and interests in using mathematics to make sense of the world (LP 4).
- given insight into how companies work to develop an environmentally responsible approach to space exploration (LP 6).
- using technology as a tool, as they employ spreadsheet and/or graphing software to perform their analysis (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



**Wayfinding:** [Iterative thinking](#) (LL.SD.2.b)

### Example 3: See a Rainbow

Students are given a problem like this to work on in groups:

*The appearance of a rainbow is a phenomenon that has captured the imaginations of people since prehistoric times. At one point, the Greek philosopher Aristotle described the rainbow in Book III of his treatise Meteorology: “The rainbow never forms a full circle, nor any segment greater than a semicircle. At sunset and sunrise the circle is smallest and the segment largest: as the sun rises higher the circle is larger and the segment smaller” (as quoted in Corradi’s A short history of the rainbow).*

*Notice that Aristotle proclaims, “the rainbow never forms a full circle.” In this assignment, you will explore this phenomenon and use tools, including [Desmos.com](#) or [GeoGebra.org](#), to construct an argument that either supports or refutes Aristotle’s statement.*

*Some materials to explore:*

- [Kamal Al-Din Al-Farisi's Explanation of the Rainbow](#)
- [Corradi history of the rainbow.pdf](#)
- [Chasing Rainbows in the Hudson Valley](#)

*Consider using sliders on [Desmos](#) or [GeoGebra](#) to explore the graph of a quadratic function and the graph of a circle.*

*One form of a quadratic function you can use is  $y = a(x - h)^2 + k$ . One form for the equation of a circle you can use is  $(x - d)^2 + (y - f)^2 = r^2$ .*

*To fit these curves to an image of a rainbow, begin by adding an image of a rainbow onto your [Desmos](#) or [GeoGebra](#) graph.*

Prepare to share your findings in a presentation, including:

- New understandings you have about rainbows from your research.
- Highlights from your exploration in fitting the graph of a quadratic function and fitting the graph of the equation of a circle to the image you used.
- How you decided which was a better fit.
- The equation of the curve of best fit and your best explanation as to what each parameter could possibly represent.

Here is one sample image you may choose to use.

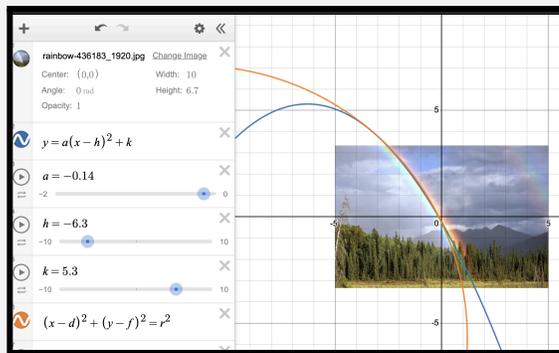
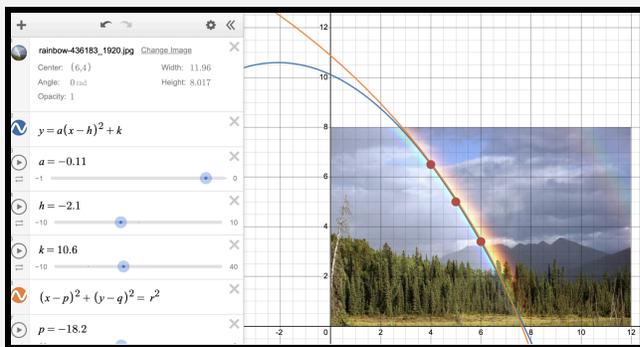


(Source: Pixabay picture: [Rainbow Bow Colors Double](#) - Free photo on Pixabay)

### Sample Learning Activity

Give students an opportunity to begin exploration independently, then invite them to work in pairs to engage in the exploration. As students work, monitor for ways they communicate their reasoning.

Some students may use features of the graphing software to focus on particular points of interest, while others may create a table of values to compare and illustrate changes they see.



Facilitate the discussion about the exploration.

Here are some questions to emphasize:

- How do they choose to place their image on the grid?
- How did they verify the focus of their work: did they select key points, color, or some other feature(s) to guide them as they explored best fit?

- *How did they determine which was a better fit?*
- *How did their research influence their approach?*

Additional questions for reflection:

- *What's promising about this approach?*
- *What questions does this approach surface for you?*
- *Do you think the model is reasonable? Why or why not?*
- *What can you take from this approach to apply to your own work?*

Give students additional time to revise their work.

In this example, students are:

- given a cognitively demanding task. It requires students to determine how to best develop an argument as to which graph is a better model for the curve of a rainbow. The task offers entry for students with a variety of prior learning activities by beginning with the use of an image of a rainbow, various tools, and resources to explore options. Through iterative cycles of work, discussion, and research, students have the opportunity to improve on preliminary models and construct a viable argument (LP 1).
- set up to have social interactions, as they work in small groups and discuss their work with each other. These interactions may happen live in a physical classroom or virtually (LP 2).
- given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening (LP 3).
- able to explore skills and dispositions around what it means to model with mathematics. As they engage in this work, students have the opportunity to develop identities and interests in using mathematics to make sense of the world (LP 4).
- able to view mathematics as a human endeavor across centuries. In exploring resources students gain insights into the ideas that shaped current understanding of rainbows. They have the opportunity to see the iterative nature of the development of scientific and mathematical ideas spanning centuries (LP 5).
- using technology as a tool, as they employ spreadsheet and/or graphing software to perform their analysis (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



**Synthesis:** [Synthesizing Information](#). (FL.ID.1.a)

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