

Annotated Examples: M101 Linear Equations: Concepts and Skills

The examples that follow are intended to illustrate how learning activities are used to support students' engagement with the content and practices outlined in this badge. They do not provide comprehensive coverage of those expectations, but rather elevate some of the learning activities less likely to be part of published curricular materials for mathematics instruction. The examples that follow were developed by the Math Badging writing team, unless otherwise specified. These are a small sample of types of learning activities that can be done with students, both in and out of a traditional classroom setting.

Example 1: Same Solutions

Which of the following equations have the same solution as $x + 3 = 5x - 4$?

- A. $x - 3 = 5x + 4$
- B. $2x + 8 = 5x - 3$
- C. $10x + 6 = 2x - 8$
- D. $10x - 8 = 2x + 6$
- E. $0.3 + \frac{1}{10}x = \frac{1}{2}x - 0.4$

Prepare to share your response.

- Explain how you could tell which equations have the same solution as the given equation without solving each one (and without substituting a value).
- Are there other equations on the list that share a common solution? Without solving, how could you tell?

(Adapted from [Illustrative Mathematics, Same Solutions?](#), 2016)

Sample Learning Activity

- What do you notice about the structure of the equations? In what ways are they the same? In what ways are they different?
- What relationships do you notice in the parts that look similar?

Think about others' approaches:

- What's promising about this approach?
- What questions does this approach surface for you?
- What can you take from this approach to apply to your own work?

Take some time to revise your work. Test additional pairs using the structure and relationship between the equations to determine which other sets of equations have the same solution.

In this example, students are:

- given a cognitively demanding task. They are asked to analyze the structure and relationships between equations to determine which equations have the same solutions. Students have multiple entry points to reason about these equations in various ways (LP 1).
- working collaboratively in pairs to complete the task, allowing for social interaction with peers. Students are also encouraged to ask their peers questions to understand each other's thinking process and revise their own thinking (LP 2).
- attending to the structure of the equations and working to make sense of the relationships between the equations. In so doing, students learn to recognize that different forms of an equation are helpful for different purposes (LP 3).
- encouraged to develop skills of expert learners, thereby developing agency, as they reflect on their approach to the prompt and incorporate new ways of thinking about this from their peers (LP 4).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



Computational Thinking: [Expressing equivalencies](#) (FL.MST.1.a)

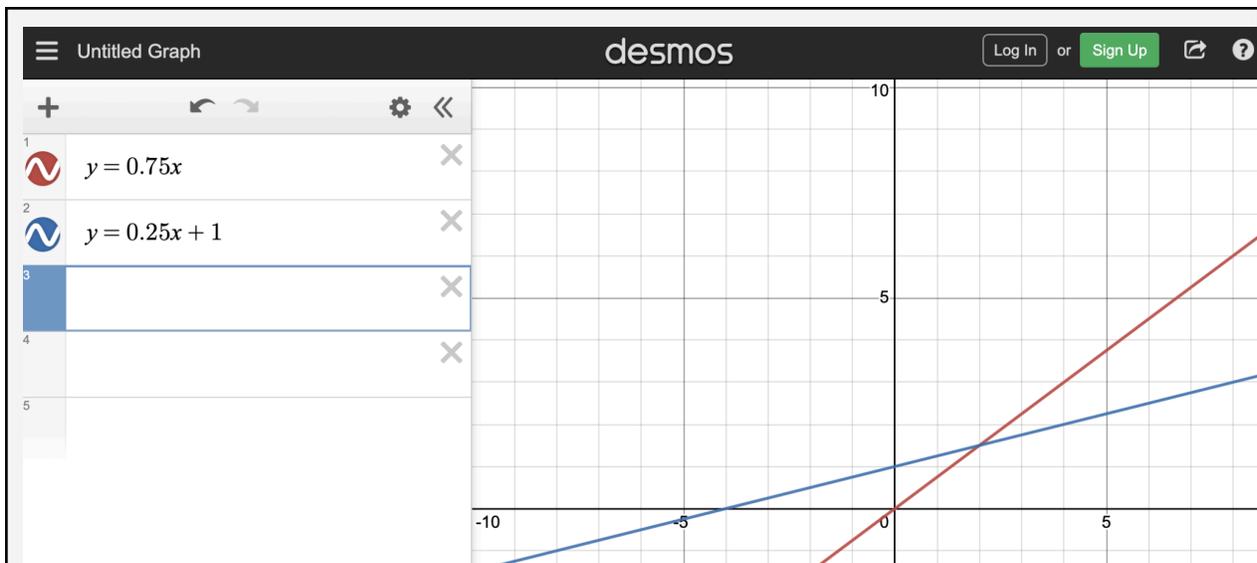
Example 2: Finding Context

Students are given an equation like this:

Consider the equation $0.75x = 0.25x + 1$

- Think about the expressions “ $0.75x$ ” and “ $0.25x + 1$.” Create a real-world situation that could be represented by each of these expressions.
- Based on this, what could x represent?
- Why would it make sense to have these two expressions be equal to one another based on the context?
- How could you use technology to create a graphical representation that shows the solution to the equation $0.75x = 0.25x + 1$?

Sample graph using technology:



- *What is the solution to $0.75x = 0.25x + 1$? What does the solution to the equation mean relative to the context you created?*
- *What is another way you could have found the solution to $0.75x = 0.25x + 1$?*

Sample Learning Activity

The goal of this task is to invite students to make sense of an alternate way of thinking about solutions to an equation in order to build towards a flexible way of working with equations. In this case, students consider what each expression of an equation could represent in terms of a system of equations. This builds conceptual understanding (LP 3).

In the first prompt, we are inviting students to give light context to the problem to help them personalize it and make concrete some of the conceptual ideas they are asked to explore.

Rephrase the prompt as necessary: *What quantity/situation could $0.75x$ represent? What quantity/situation could $0.25x + 1$ represent?*

Consider selecting a context or two for reflection. Questions worth exploring are:

- *What about this context makes sense to have two expressions that are equal to each other?*
- *What about the context you developed makes sense? What, if anything, needs to be modified so that it makes sense?*

After this discussion, students should be given time to refine the context they developed.

Later, in the exploration, we want students to notice that this equation ($0.75x = 0.25x + 1$) is in terms of x and its solution is the x -coordinate of the point of intersection of the graphs of $y = 0.75x$ and $y = 0.25x + 1$. This conversation can be expanded to examine the nuanced difference between the solution to $0.75x = 0.25x + 1$ and the solution to the system of equations defined by $y = 0.75x$ and $y = 0.25x + 1$.

This work can be expanded by asking students to also represent equations with one solution, no solution, or many solutions graphically, as well as extend their exploration to include inequalities.

During the discussion of their work, ask students to reflect and share on these questions:

- *What does it mean to solve an equation? How about a real-life problem?*
- *What is helpful about considering the graph of the system of equations?*
- *How might you use what you learned through this experience in other settings?*

In this example, students are:

- given a cognitively demanding task. They are asked to consider an alternative way of thinking about an equation, develop a context that could be represented by the equation, and interpret the meaning of the solution (LP 1).
- working collaboratively in pairs and sharing their findings with the class. Students are encouraged to ask questions of their peers to better understand the thinking process (LP 2).
- invited to give a context to the mathematical expressions to support their ability to communicate their conceptual understanding of the meaning of the solution to a mathematical equation instead of focusing on the procedural steps for solving. Students also work to reason about a specific equation in terms of a system of equations, making way for a deeper understanding of solving equations (LP 3).
- invited to give context to the mathematical expressions, giving students an opportunity to represent their interests and experiences in a mathematical way (LP 4).
- leveraging technology to examine a graphical representation of given expressions and to support their conceptual understanding of the quantities represented by the expressions, the meaning of the variable(s), constants, and the meaning of any solution(s) (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



Computational Thinking: [Expressing equivalencies](#) (FL.MST.1.a)

Example 3: Rainy Day Fund

Students are given a problem like this:

Sandra works as a fitness manager at her local gym. She earns enough to pay for her living expenses and save \$2000 per month to add to her savings account. She has already saved \$12,000 for emergencies, which she estimates to be enough for 6 months of expenses.

- Write an equation to represent the balance in Sandra's account after x months.*
- Graph this equation using [Desmos.com](#) or [geogebra.org](#).*
- Describe the key features of the graph. Then, explain them in the context of Sandra's savings.*

Next, explore changes to the equation and graph as Sandra encounters different scenarios. Use sliders by typing in " $y = m(x) + b$ " and add "all" sliders on [Desmos](#) or refer to this video to see how to do it on [GeoGebra](#).

1. How does the equation and graph change if Sandra gets a raise and her salary increases? Explain how you know.
2. How does the equation and graph change if Sandra changes the amount she saves every month? Explain how you know.
3. What connections do you see between the equation you created in A above and the ones you've been exploring?

The following problem provides a nice extension.

Students are given a scenario like this:

The math class was asked to solve $4x + 12 < 6x + 18$.

One student answered as follows:

$$4(x + 3) < 6(x + 3)$$

$$4 < 6$$

Therefore, there is no solution.

Decide if you agree or disagree with the student's solution and explain your reasoning.

Sample Learning Activity

Provide time for students to consider the question posed and independently develop a response. Have students write their response including their justification. Next, have partner pairs share their written justification and critique each other's response, allowing for students to defend their response. Provide time for students to revise their written justifications using the information gained during their partner-sharing time. While observing student discussions, identify varying solution methods to be presented during the whole group sharing. Have selected students present their responses—consider using a document camera for students to show actual work—and explain their justification.

Some initial justifications that may surface:

Gonzalo: *"Well if you look at it, you can tell 18 is bigger than 12. And $6x$ is always more than $4x$. So, that means this is true. So, yeah, 6 is greater than 4. No matter what."*

Nuri: *"I don't agree. The answer is right here. [Student points to $4 < 6$] I also see the same thing here [Student points to expression $x + 3$ on both sides of the inequality]. But they are divided by different things, and you can't do that."*

Stella: *"I don't agree. I think it works for any number because I worked it out*

$$4x + 12 < 6x + 18$$

$$4x < 6x + 6$$

$$0 < 2x + 6$$

See, 6 is always greater than 0."

While sharing justifications, ask probing questions to make students' thinking visible, such as:

- *I noticed you compared terms from each side of the inequality, 18 and 12 and $6x$ and $4x$. Can you say more about that? Why does it make sense to look at these pairs of terms?*
- *What do you mean when you say, “no matter what”?*
- *I noticed you recognized that they had the same expression, $x + 3$, on both sides of the inequality at one point. Why do you think this student took time to factor each expression? What might be helpful about this form?*

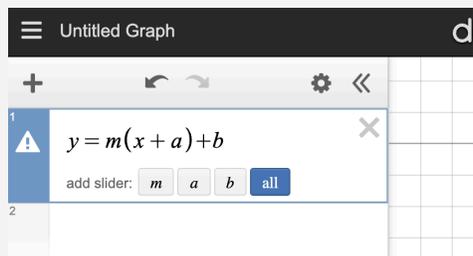
As discussion takes place, invite classmates to join the discussion. Ask:

- *What about this reasoning makes sense?*
- *Is there anything you disagree with?*
- *What questions surface for you?*
- *Are there some values of x where you are convinced this statement is absolutely true? What values might not make this inequality true?*
- *How might the factored form of the inequality help us confirm these ideas?*

Provide time for students to revise their work.

Extend the Learning:

Next, let's build on this conversation and focus on the connection between equations and graphs. Select one of the following equations: $y = m(x + a) + b$ or $y = m(x - a) + b$. Then, add “all” sliders to your chosen equation (see image).



- *Use the sliders to explore the changes that occur in the graph when the values of m , a , and b are manipulated.*
- *How would you explain the impact that manipulating the value of a has on the graph of the linear equation?*

Sample Learning Activity

Give students an opportunity to independently begin exploration, then invite them to work in pairs to engage in the exploration. As students work, monitor for the ways they communicate their reasoning. Some students may use features of the graphing software to focus on particular points of interest, while others may create a table of values to compare and illustrate the changes they see.

Facilitate the discussion about the exploration.

Here are some points to emphasize:

- Highlight connections between the context, features of the graph (slope and y-intercept), and impact on the graph as constraints and assumptions change. For example, if Sandra gets a raise, the line has a greater slope and the line is steeper. The rate of change is also greater.
- Encourage students to compare the structure of the resulting equations, when parameters changed, with the structure of the original equation they determined in A.
- Invite students to explore and describe the impact of having this starting equation: " $y = m(x + a) + b$ " or " $y = m(x - a) + b$ ".

Give students additional time to revise their work.

In this example, students are:

- given a cognitively demanding task. They are asked to explore and explain how changes in parameters impact the graph of a linear equation, as well as how those changes connect to a real-life context (LP 1).
- working individually and collaboratively as they make progress through this exploration. Students are encouraged to ask questions to their peers to understand one another's thinking process (LP 2).
- given a task that centers reasoning with multiple opportunities to make sense of the impact of manipulating parameters in a linear equation on its graph. Students build in-depth understanding as they work to justify these changes (LP 3).
- given access to a graphing tool to aid in their understanding of and reasoning with multiple representations of the equation of a line (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ Competency:



Mathematical Modeling: [Modeling](#) (FL.MST.2.a)

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