

M104: Modeling with Exponential Functions

[Illustrative Mathematics–Algebra 1 Unit 5](#)

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage structure of the curriculum to give space for students to **have agency in their learning (LP4)**. This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- While most of the modeling cycle is well represented, there are few opportunities for students to formulate their own questions or to identify the variables of interest in a given situation or data set. The use of [Math Language Routine 5: Co-Craft Questions](#) may be useful in building these opportunities into some of the existing modeling tasks within the unit, particularly some of the more complex tasks toward the end of the unit. This is necessary to fully meet [Content and Practice Expectation 104a](#).
- The educator will need to bring in space to attend to [Indicator 104.h](#). Consider using [this template](#) to offer students space to reflect and document their thinking at key points in time. Spaces where we see an opportunity for this are indicated by a triangle ▲.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one’s thinking has excellent value in solidifying one’s knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students’ identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental

challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you...
104.a: Engage in the modeling cycle (performance assessment).	i. engage with the full modeling cycle (problem, formulate, compute, interpret, validate, revise as necessary, report).
104.b: Interpret exponential functions that arise in applications in terms of the context.	i. select two different data points that illustrate something interesting or important about the context you are modeling and describe what these points tell you about the topic.
	ii. use a model to compute output values and then interpret the values in the context of the problem.
	iii. describe the domain of the exponential function, including anything noticed about values that are not part of the domain or why the shape of the model might change for particular values of the domain.
	iv. describe the contextual meaning of the average rate of change between two points for an exponential function.
104.c: Analyze exponential functions using different representations.	i. create an exponential graph to show a particular context and describe important values that help to understand something about the problem you are exploring.

Content and Practice Expectations	Indicators Choose an artifact where you...
	ii. explain how the parameters of an exponential model relate to its graph.
104.d: Build an exponential function that models a relationship between two quantities.	i. identify the variables of interest in a given situation or data set that are interesting and that demonstrate an exponential relationship. ii. build a function that models the exponential relationship between two quantities in a situation.
104. e: Construct and compare linear and exponential models, solve problems, and draw conclusions.	i. distinguish between situations that can be modeled with linear functions and those that can be modeled with exponential functions. ii. provide insight into why some relationships are linear and others are exponential or explain why, in terms of a situation being modeled, an exponential function is a better model than a linear function.
104.f: Interpret expressions for exponential functions in terms of the situation they model.	i. describe the meaning of the parameters for an exponential function in terms of the context.
104.g: Summarize, represent, and interpret data on two quantitative variables for linear and exponential model fits.	i. choose a particular representation of your data to communicate something important to a particular person or audience and explain why you made that choice. In this badge, students are encouraged to investigate patterns of association in bivariate data, which includes the informal description of the fit of the curve and addresses the usefulness of the model for the particular context.
104.h: Understand the relevance of modeling with exponential functions.	i. recognize something that surprised you or that you learned about the topic through exponential modeling of data. Share what surprised you or what you learned. ii. highlight something you are most proud of in your learning about exponential modeling. iii. explain how modeling can be used to answer important questions.
104.i. Use functions of exponential type to	i. determine one or more output values for specific inputs to the function.

Content and Practice Expectations	Indicators Choose an artifact where you...
determine values of interest in a real-world problem.	

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19	L20	L21	MT	
104a.i																							x
104.b.i		x	x	x	x	x	x		x	x		x	x		x								
104.b.ii		x	x	x	x	x	x	x	x	x		x	x		x	x	x		x			x	x
104.b.iii					x	x	x		x														x
104.b.iv										x		x			x								
104.c.i			x		x	x	x	x	x			x			x					x		x	
104.c.ii			x	x	x	x	x	x	x	x		x			x					x		x	
104.d.i									x														
104.d.ii				x			x	x	x				x		x	x	x		x			x	x
104.e.i		x		x	x																x	x	x
104.e.ii		x	x	x	x										x	x				x	x	x	x
104.f.i			x		x	x	x		x	x		x			x		x		x				

104.g.i							x		x							x					x	x
104.h.i					▲	▲	▲	▲	▲	▲		▲	▲		▲	▲	▲		▲		▲	▲
104.h.ii					▲	▲	▲	▲	▲	▲		▲	▲		▲	▲	▲		▲		▲	▲
104.h.iii					▲	▲	▲	x	▲	▲		▲	▲		▲	x	x		x		x	x
104.i.i			x	x	x	x	x	x	x	x		x	x			x			x		x	x

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.1	D Not needed to enter Lesson 5.2. Genie context extension not aligned with LPs.			Context: Bacteria growth, genie wishes Math focus: Linear and non-linear growth basics, review of grades 6,7,8
5.2	M Prioritize Activity 5.2.3 to give students an opportunity to see math as relevant (LP6), especially if students express an interest in use of social media. Doing so may mean skipping Activity 5.2.2.	104.b.i 104.b.ii 104.e.i 104.e.ii	LP3, LP6	Context: Social media Math Focus: Understanding the difference between exponential and linear
5.3	K	104.bi 104.b.ii 104.c.i 104.c.ii 104.e.ii 104.f.i	LP3	Context: Bacteria growth Math Focus: Multiple ways to represent exponential relationships

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		104.i.i		
5.4	M Prioritize activity 5.4.3, and build ideas from activity 5.4.2 into discussion to optimize space for LPs.	104.bi 104.b.ii 104.c.ii 104.d.ii 104.e.i 104.e.ii 104.i.	LP2, LP3, LP6, LP7	Context: Value of car → intro decay Math Focus: Growth factor, tables, expressions, graphs
5.5	K	104.b.i 104.b.ii 104.b.iii 104.c.i 104.c.ii 104.e.i 104.e.ii 104.f.i 104.i.i	LP3, LP4, LP5, LP6, LP7	Contexts: Containment of algae bloom, insulin in body Math Focus: Continues thread of exponential decay, engages students in aspects of mathematical modeling
5.6	K	104.b.i 104.b.ii 104.b.iii 104.c.i 104.c.ii 104.f.i 104.i.i	LP3, LP4, LP6, LP7	Context: Value of cell phones Math Focus: Analyzing how quantities change, finding growth factor Card sort provides opportunity to synthesize learning

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.7	M Prioritize activities 5.7.2 and 5.7.3.	104.b.i 104.b.ii 104.b.iii 104.c.i 104.c.ii 104.d.ii 104.f.i 104.g.i 104.i.i	LP3, LP4, LP5, LP6, LP7	Context: Coral reef Math Focus: Introduction to negative exponents
5.8	M Prioritize activities 5.8.1, 5.8.3, and 5.8.4,	104.b.ii 104.c.i 104.c.ii 104.d.ii 104.f.i 104.g.i 104.h.iii 104.i.i	LP2, LP3, LP6, LP7	Context: Mold growth, bacteria growth, medicine in human body Math Focus: Constructing and using tables, equations, and graphs to represent functions
5.9	M Prioritize activities 5.9.2 and 5.9.3. MLR4: Info Gap activity key to some of the CPEs and LPs.	104.b.i 104.b.ii 104.b.iii 104.c.i 104.c.ii 104.d.i 104.d.ii 104.f.i	LP2, LP3, LP6, LP7	Context: Solar costs (decay , paper folding, smartphone sales Math Focus: Interpreting and reasoning about exponential functions

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		104.g.i 104.i.i		
5.10	K	104.b.i 104.b.ii 104.b.iv 104.c.ii 104.f.i 104.h.iii 104.i.i	LP1, LP3, LP6	Context: Coffee shop, follow up on solar cost Math Focus: Review of constant rate of change to compare to average rate; calculate avg rates (including decay) in graph, table, and equation
5.11	D Not needed to enter the next lesson. Optional activity requires materials (types of sports balls), yet the motivation/reason to model ball bounce is not connected to real relevance.			Context: Bounce height of balls Math Focus: Mathematical modeling
5.12	K	104.b.i 104.b.ii 104.b.iv 104.c.i 104.c.ii	LP3, LP7	Context: Bank accounts Math Focus: Exponential decay, exponential equations to graphs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
		104.f.i 104.i.i		
5.13	K	104.b.i 104.b.ii 104.d.ii 104.i.i	LP1, LP3, LP6	Context: Value of a computer, mold growth Math Focus: Exponential graphs to equations
5.14	D Not needed to enter next lesson, listed as “optional” by authors.			Context: Costs of scooters, bikes, etc., taxes and sales Math Focus: Review of grade 7 concept of percent increase
5.15	K	104.b.i 104.b.ii 104.b.iv 104.c.i 104.c.ii 104.d.ii 104.e.i 104.f.i	LP1, LP2, LP3, LP6	Context: Interest on borrowed amounts Math Focus: Using exponential expressions to model repeated interest calculations

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.16	K	104.b.ii 104.d.ii 104.e.ii 104.g.i 104.h.iii 104.i.i	LP1, LP2, LP3	Context: Resizing images, earning interest Math Focus: Distinguishes between successive percent changes over time versus one overall percent change
5.17	M Prioritize activity 5.17.2.	104.b.ii 104.d.i 104.f.i 104.h.iii	LP3, LP6	Context: Interest on earnings, how credit cards work, financial literacy Math Focus: Compounding interests (compounding intervals)
5.18	D Not aligned to expectations of this badge and not needed for coherence.			Context: Population growth, interest on earnings Math Focus: Properties of exponents
5.19	K The structure of this lesson aids in developing a way to approach open-ended prompts, similar to most open versions of	104.b.ii 104.c.i 104.c.ii 104.d.ii 104.e.ii 104.h.iii 104.i.i	LP1, LP3, LP6, LP7	Context: Compound vs. simple interest Math Focus: Comparing growth of linear and exponential functions

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	the modeling prompts offered.			
5.20	K	104.e.i 104.e.ii	LP1, LP2, LP3	Context: Purely mathematical Math Focus: Rates of change, what makes something linear vs exponential
5.21	K	104.b.ii 104.c.i 104.c.ii 104.d.ii 104.e.i 104.e.ii 104.g.i 104.h.iii 104.i.i	LP1, LP2, LP3, LP6, LP7	Context: Population [cities → world] Math Focus: Using linear and exponential functions for modeling
Modeling Task- <u>Critically Examining National Debt</u>	K Two versions are given: one in which data is provided, and one in which other students do research to find data to use.	104.a.i 104.b.ii 104.b.iii 104.d.ii 104.e.i 104.e.ii 104.g.i 104.h.iii 104.i.i	LP1, LP2, LP3, LP6, LP7	Context: US national debt Math Focus: Modeling with linear and exponential functions

