

M205: Modeling with Trigonometric Functions

[Illustrative Mathematics–Geometry Unit 7: Circles](#)

[Illustrative Mathematics–Algebra 2 Unit 6: Trigonometric Functions](#)

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4)**. This can be accomplished by learning about students’ interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum’s mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one’s thinking has excellent value in solidifying one’s knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you...
205.a: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	i. use the definition of a radian to explain radian measure.
	ii. explain what an angle given with a measure in degrees would be in radians.
205.b: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers.	i. explain how radians lead to a dimensionless number that makes it possible to extend the domain of trigonometric functions.
	ii. use the unit circle to justify the value of sine, cosine, or tangent for an angle that is greater than 90 degrees.
205.c: Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	i. describe the symmetry of a trigonometric function.
	ii. use the unit circle to explain why a trigonometric function has symmetry.
	iii. describe the periodicity of a trigonometric function.
	iv. use the unit circle to explain why a trigonometric function is periodic.
205.d: Analyze trigonometric functions using different representations.	i. create the graph of a trigonometric function.
	ii. identify key features of the graph of a trigonometric function, such as its period, amplitude, and frequency, and connect these features back to the context of the problem.
	iii. explain the relationship between the parameters of a trigonometric function and its graph.
205.e: Use trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	i. create a trigonometric function to model a real-world situation.
	ii. use a trigonometric function to answer a question about a real-world situation.

Unit Overview

	<u>L1</u>	<u>L2</u>	<u>L3</u>	<u>L4</u>	<u>L5</u>	<u>L6</u>	<u>L7</u>	<u>L8</u>	<u>L9</u>	<u>L10</u>	<u>L11</u>	<u>L12</u>	<u>L13</u>	<u>L14</u>	<u>L15</u>	<u>L16</u>	<u>L17</u>	<u>L18</u>	<u>L19</u>	<u>L20</u>	<u>L21</u>	<u>MT</u>
205.a.i	X				X																	
205.a.ii		X			X																	
205.b.i				X		X	X	X	X				X									
205.b.ii				X		X	X	X	X			X	X									
205.c.i						X	X	X	X				X									
205.c.ii			X				X	X	X				X									
205.c.iii			X							X		X	X	X				X	X	X		
205.c.iv			X									X	X	X								
205.d.i											X	X	X	X	X	X	X	X	X	X	X	X
205.d.ii											X	X	X	X	X	X	X	X	X	X	X	X
205.d.iii											X	X	X	X	X	X	X	X	X	X	X	X
205.e.i									X						X	X				X	X	X
205.e.ii									X						X	X		X		X	X	X

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
G.7.11 (Use Lesson 11 from Unit 7 of Geometry Course)	K	205.a.i	LP2, LP3, LP5	Context: Purely mathematical Math focus: Defining radian measure as the ratio of the arc length to the circle's radius
G.7.12	K	205.a.ii	LP2, LP3	Context: Pie coloring Math focus: Understanding angles measured in radians
A2.6.1 (Transition to Algebra 2 Unit 6 Lessons)	K The content lays a foundation for identified CPEs.	205.c.ii 205.c.iii 205.c.iv	LP2, LP3, LP6	Context: Ladybug on clock's second hand Math focus: Using Pythagorean theorem to find coordinates of points on a circle centered at origin; introduces the concept of periodic functions

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
6.2	K The content lays a foundation for identified CPEs.	205.b.i 205.b.ii	LP2, LP3	Context: Purely mathematical Math focus: Using trigonometry to express the coordinates of a point in quadrant 1, one unit away from origin
6.3	K	205.a.i 205.a.ii	LP1, LP2, LP3, LP6	Context: Bicycle wheel Math focus: Unit circle—radian angle measurements and defining points by coordinates or angle of rotation
6.4	M Consider asking students to describe the connection between the coordinates to trig functions within this lesson to address CPE 205.b.	205.b.i 205.b.ii 205.c.i	LP2, LP3	Context: Purely mathematical Math focus: Unit circle—finding angles and estimating coordinates
6.5	M Teaching note: the idea of symmetry is implied but not directly stated. Be sure to discuss symmetry of the trig	205.b.i 205.b.ii 205.c.i 205.c.ii	LP2, LP3	Context: Purely mathematical Math focus: Pythagorean identity—calculating values of coordinates given one; direct work with unit circle

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	function to address CPE 205.c.i.			
6.6	K	205.b.i 205.b.ii 205.c.i 205.c.ii	LP2, LP3, LP5	Context: Purely mathematical Math focus: Pythagorean identity—finding the values of cosine, sine, and tangent if one is known
6.7	M Components of this lesson build a foundation for CPE 205.e, since students work with trig expressions, not functions.	205.b.i 205.b.ii 205.c.i 205.c.ii 205.e.i 205.e.ii	LP1, LP2, LP3, LP6	Context: Clock/ferris wheel Math focus: Using cosine and sine to figure out information about points rotating in circles
6.8	K	205.c.iii	LP2, LP3, LP5	Context: Periodic functions in context (distance from startline, temperature, and population) Math focus: Understanding that the graph of a periodic function can look like a wave whose outputs repeat between the same max and min values

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
6.9	K	205.d.i 205.d.ii 205.d.iii	LP1, LP2, LP3, LP7	Context: N/A—use of applet to obtain coordinates, given an angle Math focus: Using coordinates of points on the unit circle to graph the cosine and sine functions
6.10	K	205.b.ii 205.c.iii 205.c.iv 205.d.i 205.d.ii 205.d.iii	LP2, LP3, LP6	Context: Windmill blade Math focus: Understanding how to find the values of cosine and sine for inputs greater than 2π radians
6.11	K	205.b.i 205.b.ii 205.c.i 205.c.ii 205.c.iii 205.c.iv 205.d.i 205.d.ii 205.d.iii	LP1, LP2, LP3, LP7	Context: Windmill; clock hand Math focus: Understanding how to find the values of cosine and sine for inputs less than 0 radians

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
6.12	K	205.c.iii 205.c.iv 205.d.i 205.d.ii 205.d.iii	LP2, LP3	Context: Purely mathematical Math focus: Understanding why the tangent function has a period of pi; understanding why the graph of tangent has asymptotes; exploration of tangent graph
6.13	K	205.d.i 205.d.ii 205.d.iii 205.e.i 205.e.ii	LP2, LP3, LP6, LP7	Context: Windmill blade Math focus: Writing trigonometric functions to represent situations with different amplitudes and midlines
6.14	M Teaching note: ask students to create equations in Activity 14.3 (Spinning Fan).	205.d.i 205.d.ii 205.d.iii 205.e.i 205.e.ii	LP1, LP2, LP3, LP6	Context: Windmill blade; spinning fan Math focus: Graphing horizontal translations of trigonometric functions; using amplitude and midline to describe a situation
6.15	K	205.d.i 205.d.ii 205.d.iii	LP2, LP3, LP7	Context: Sound waves; matching activity Math focus: Identifying midline, amplitude, and horizontal translations given a graph or equation

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
6.16	K	205.c.iii 205.d.i 205.d.ii 205.d.iii 205.e.ii	LP1, LP2, LP3, LP6	Context: Ferris wheel Math focus: Finding the period using an equation or graph
6.17	K	205.c.iii 205.d.i 205.d.ii 205.d.iii	LP1, LP2, LP3, LP5	Context: Purely mathematical Math focus: Creating an equation of a trigonometric function using information about its graph
6.18	K	205.c.iii 205.d.i 205.d.ii 205.d.iii 205.e.i 205.e.ii	LP1, LP2, LP3, LP5, LP6	Context: Bike wheel; carousel Math focus: Representing a circular motion situation using a graph and an equation
6.19	K	205.d.i 205.d.ii 205.d.iii 205.e.i 205.e.ii	LP1, LP2, LP3, LP5, LP6, LP7	Context: Visibility of the moon Math focus: Creating a model of data that is approximately periodic and using the model to make predictions

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
Modeling Task- <u>Swept Away</u>	K Two versions are given—the second version has a few more questions analyzing the situation before asking students for a model. Consider using local data to increase degree of relevance.	205.d.i 205.d.ii 205.d.iii 205.e.i 205.e.ii	LP1, LP2, LP3, LP5, LP6, LP7	Context: Tides in Boston Math focus: Modeling with trigonometric functions