

M202: Rational Exponents and Complex Numbers

[Illustrative Mathematics—Algebra 2 Unit 3-Complex Numbers and Rational Exponents](#)

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4)**. This can be accomplished by learning about students’ interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum’s mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one’s thinking has excellent value in solidifying one’s knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you...
202.a: Reason about and extend the properties of exponents to rational exponents.	i. explain how the meaning of rational exponents follows from extending the properties of integer exponents to those values, and how this definition leads to notation for radicals in terms of rational exponents.
	ii. rewrite expressions involving radicals and rational exponents using the properties of exponents.
202.b: Reason about and perform operations with complex numbers.	i. demonstrate understanding that there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
	ii. use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
202.c: Analyze and use complex numbers in polynomial equations.	i. solve quadratic equations with real coefficients that have complex solutions.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	L19
202a.i			X	X	X														
202.a.ii			X	X	X														
202.b.i							X			X	X		X	X					
202.b.ii												X	X	X					

202.c.i																	X	X	X	
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Note: Columns in gray indicate a recommendation to delete the lesson.

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.1	D This lesson is listed as optional by authors. This lesson reviews exponent rules that were developed in grade 8 where the exponents were integers. Use this lesson as needed.			Context: Purely mathematical Math focus: Properties of exponents, where exponents are integers; and evaluating expressions with integer exponents
3.2	D This lesson is listed as optional by authors. This lesson reviews square and cube roots. Use this lesson as needed.			Context: Purely mathematical; use of area and volume as context Math focus: Calculating square and cube roots

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.3	K	202.a.i 202.a.ii	LP1, LP2, LP3, LP5, LP7	Context: Purely mathematical Math Focus: Writing square and cube roots as rational exponents
3.4	M Prioritize Activity 4.3. One way to do this is to embed some of the questions from Activity 4.2 within the discussion of Activity 4.3.	202.a.i 202.a.ii	LP1, LP3, LP7	Context: Purely mathematical Math Focus: Using graphs to estimate the value of expressions involving positive rational exponents
3.5	K	202.a.i 202.a.ii	LP2, LP3, LP7	Context: Purely mathematical Math Focus: Using graphs to estimate the value of expressions involving negative rational exponents
3.6	D No connection to CPEs.			Contexts: Purely mathematical Math Focus: Understanding that the square root symbol means the positive square root; seeing solutions to $x^2 = a$ and $y = \sqrt{x}$ in graphs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.7	M Prioritize Activities 3.7.2 and 3.7.3.	202.b.i	LP1, LP2, LP3	Context: Purely mathematical Math Focus: Reason about solutions to equations involving squares and square roots
3.8	D No connection to CPEs.			Context: Purely mathematical Math Focus: Solving cube root and cubic equations algebraically
3.9	D This lesson is listed as optional by authors. It is an opportunity for extra practice in solving a variety of radical equations.			Context: Purely mathematical Math Focus: Solving equations with radicals in them
3.10	K	202.b.i	LP1, LP3, LP5	Context: Purely mathematical Math Focus: Representing imaginary numbers visually with the imaginary number line

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.11	K	202.b.i	LP1, LP3, LP5, LP6	Context: Purely mathematical Math Focus: Using i to solve quadratic equations and representing solutions to equations using i and the complex plane
3.12	M Prioritize Activity 12.2. Activity 12.3 sets the stage for the next lesson—consider using it at some point to introduce Lesson 13.	202.b.ii	LP1, LP2, LP3	Context: Purely mathematical Math Focus: Using the complex plane to add complex numbers; calculating powers of imaginary numbers
3.13	K	202.b.i 202.b.ii	LP1, LP2, LP3	Context: Purely mathematical Math Focus: Multiplying complex numbers
3.14	K	202.b.i 202.b.ii	LP3	Context: Purely mathematical Math Focus: Practicing adding, subtracting, and multiplying complex numbers

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.15	D Recommend to use if seeing that extra time on this will support students. Good lesson to engage students in reasoning about complex numbers as a way to synthesize learning.			Context: Purely mathematical Math Focus: Finding real and imaginary parts of complex numbers if given enough information about the numbers and their product
3.16	K	202.c.i	LP1, LP3, LP4	Context: Purely mathematical Math Focus: Solving quadratic equations by completing the square or by using the quadratic formula
3.17	K Please note Activity 17.4 is listed as optional in this lesson. This allows you to prioritize 17.2 and 17.3.	202.c.i	LP1, LP3	Context: Purely mathematical Math Focus: Solving quadratic equations by completing the square

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
3.18	K	202.c	LP1, LP3	<p>Context: Purely mathematical</p> <p>Math Focus: Finding complex solutions to quadratic equations by using the quadratic formula</p>
3.19	D Listed as optional by authors.			<p>Context: Using the value under the radical $b^2 - 4ac$ to determine the number of real solutions</p> <p>Math Focus: Determining how many real solutions a quadratic equation has</p>