

Annotated Examples: M152 Reasoning and Proof through Congruence

The examples that follow are intended to illustrate how the learning principles are used to support students' engagement with the content and practices outlined in this badge. These examples do not provide comprehensive coverage of those expectations, but rather elevate some of the learning principles that are less likely to be part of published curricular materials for mathematics instruction. The examples that follow were developed by the Math Badging writing team, unless otherwise specified. These are a small sample of types of learning activities that can be done with students, both in and out of a traditional classroom setting.

Example 1: Name That Logo

Students are given a problem like this:

In the field of marketing, it is crucial to create a brand that stands out from the rest! Think about your favorite brand logos. What about those logos make them memorable?

One of the ways that companies capture their audience is through symmetry. Symmetrical patterns are said to give the human brain a sense of familiarity and order, making it easier to remember products. Take a moment to explore the article [Why Do We Get So Much Pleasure From Symmetry?](#) from the website [howstuffworks](#) to learn more.

- 1.** Name a brand that comes to mind for you initially. Explain any symmetry you see represented in the brand's logo.
- 2.** Select two company logos with symmetry, one with line symmetry and one with rotational.
 - a.** Make an argument for which logo is more aesthetically pleasing in terms of the type of symmetry.
 - b.** Work to give detailed explanations wherever possible. For example, for the logo with rotational symmetry, identify and explain the degree of the rotation.
- 3.** Let's explore a company logo that took a different approach by not using symmetry in its design.
 - a.** Find and select a logo you like that fits this description. What is something about this logo you appreciate?
 - b.** Redesign the logo so it is visually appealing **and** has rotational symmetry.
 - c.** Explain your chosen degree of rotation for your new logo. What is the degree of rotation you decided to use? In what ways does this honor what you appreciate about the logo or company?

Sample Learning Activity

Launch this lesson with a "Which one doesn't belong?" warm up activity. Create a four-image comparison of logos that are a mix of symmetrical and non-symmetrical. Give students silent thinking time to create an argument around their choice. Have them share out in groups or to the whole class. Consider using [Four](#)

Corners to have students share their arguments with others who selected the same choice before sharing with the whole class.

Questions that can be posed during share out:

- *Which logos have line symmetry?*
- *Which logos have rotational symmetry? What do you think is the degree of rotation? What did you pay attention to in the logo to land on that response?*
- *What other logos come to mind? Which of these have symmetry? How can you tell?*

Give students time to research logos as they work to respond to Question 2. Invite them to research logos in order to identify two logos: one with line symmetry and another with rotational. Monitor their progress and select student work that illustrates understanding of symmetry. Have students share out and explain identified symmetry for each logo selected. Encourage them to ask questions to each other. Consider using the following questions:

- *What is clear to you from this explanation?*
- *What is something you want to hear more about?*
- *What additional questions surface for you?*

Following the share out, give students an opportunity to further develop, modify, or revise their work.

Give students time to research logos as they work on Question 3 of the task. Invite them to identify a logo that does not utilize symmetry in its design. Give students time to begin exploration independently, then invite them to work in pairs to engage in the design process by selecting one of their logos to work together to make it more aesthetically pleasing. Give students the option to utilize tools and web-based applications, like tracing paper, [Desmos](#), [GeoGebra](#), Canva, or Photoshop as they redesign the logo of their choice.

Consider concluding this exploration by having students participate in a gallery walk to showcase their designs.

In Example 1, students are:

- given a cognitively demanding task requiring them to reason about transformations. The learning activity is structured to give all students access by inviting them to select which one does not belong and builds up to engage every student in the redesign of a logo using their understanding of transformations (LP 1).
- working individually and collaboratively in pairs to complete the task, allowing for social interaction among students. They are also encouraged to ask questions to each other to better understand the thinking process (LP 2).
- building conceptual understanding through reasoning, as they identify symmetry in logo designs and create a logo with rotational symmetry (LP 3).
- given space to develop agency, as they choose their solution pathways and reflect on how their approach might change based on engagement with their peers' work. Students also reflect on how the designs they create reflect what they value (LP 4).
- given insight into how mathematics is used in the field of marketing by understanding how symmetry can enhance logo designs (LP 6).

- using technology as a tool, as they employ tracing paper and/or web-based apps to identify the degree of rotation and redesign a logo (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Economics:** [Economic Forces](#) (FK.SS.3.b)

Example 2: Mathematics In Art

Suggested warm up:

Using an online quizzing game, have students participate in a logo guessing quiz.

Students are given a problem like this:

Part 1: Mathematics in Art

Artists like Kiayani “Kay” Douglas use mathematics as they design their artwork. In this task you will have the opportunity to learn more about Ms. Douglas and her art, as well as time and space to create your own artwork using some of the mathematical ideas she shares with us, specifically tessellations.

Watch this interview with Ms. Douglas about her Portraits Collection and the use of tessellations:
<https://bit.ly/3Rh7tJM>.

1. What intrigued you about what Ms. Douglas shared? What questions did it surface for you?
2. What might you want to learn or explore further in preparation to create your own artwork?
3. What insights from Ms. Douglas' video do you want to incorporate into your artwork?

Part 2: Create Your Own Artwork

In the video, we learned how Ms. Douglas designed the tessellations present in her portrait collection by working with a square. We also learned that working with tessellates and other tools, like ink and colored pencils, can be used to create images on that piece, much like Ms. Douglas' outline of Africa and the power fist.

For this task you will explore and design a tessellating piece for your final work of art.

1. Select a polygon to base your template (tessellating piece) on: a square or other polygon (triangle, rectangle, or hexagon).
2. Explore ways to modify the polygon to create a piece that tessellates.
3. Iterate your piece to create a sample tessellation using your template. What transformations are in play? What images can you see surfacing? What might you draw on it to create the image you want to share?

4. Repeat Steps 1-3 until you find the shape you will use as the template of the tessellating piece in your artwork.

Create your work of art. Consider how you will use color, space, and other elements to design artwork that you're proud of. Upon completing your artwork, write or record a statement addressing the following questions:

- Tell us a little about yourself. What is something you learned about yourself in doing this work?
- Tell us about your art piece. What inspired this work? How would you describe the composition and elements of this piece? How would you describe what we're looking at?
- Tell us about your tessellating piece. How did you construct the template? How did you design it to take on the image(s) it currently holds? What transformations are evident in your template?
- Tell us about the tessellation. How does your tessellating piece fill the space? How did you use this space? How would you describe the transformations present in your tessellation?

Sample Learning Activity

Part 1: Mathematics in Art

Begin by inviting students to share what they know about how mathematics may surface in art. Listen for students sharing elements such as shapes, symmetry, perspective, patterns, and proportion. Use these elements to set the stage for listening to Kiayani “Kay” Douglas’ video. Share the video. Facilitate a discussion and have students reflect on what they heard.

Highlight the use of transformations to form the background for the shared artwork. Encourage students to visit Ms. Douglas’ website, <https://www.iamkaydouglas.com/>, to explore and look at the images she highlighted more closely.

Move the conversation towards coming to a shared understanding of what a tessellation is and establish comfort with identifying the tessellating piece. Encourage students to identify and share where tessellations might be found in everyday life, e.g., tile work on floors and walls, some wallpaper patterns, and rugs.

Part 2: Create Your Own Artwork

Give students individual work time to develop a template for the tessellating piece. You may opt to use pattern blocks as a precursor to deepen an understanding of how polygons work to tessellate a plane. For example, we can tile using one polygon—tessellating piece—or we may use multiple pieces to tile a plane. Pattern blocks can make that visible. This exploration can lead to a list of polygons that students may want to consider as the basis of their tessellation.

Provide students multiple copies of various polygons they can use to explore different options, as illustrated here: [A Simple Method For Creating Tessellations From Rectangles](#) or [Tessellations](#). As students work, encourage them to explore the impact of translating, reflecting, and rotating cut-outs on overall tessellation (Steps 2 and 3). Encourage students to work in pairs as they create their template to share and

discuss their progress with each other. Make dynamic geometric applications, like [Desmos](#) and [GeoGebra](#), available for students to use to create their template and tessellation.

Monitor and select student work to share with the class. Ask selected students to share how they have approached the task of creating the template for the tessellating piece. Consider asking:

- *How did you go from the original polygon to the shape we see now?*
- *What transformations did you use?*
- *How would you describe what you see happening as you tessellate your piece? What transformations do you see in play?*

Encourage students to engage with and make sense of each other's designs, using these questions:

- *What do you appreciate about this approach?*
- *What questions does this approach surface for you?*
- *What can you take from this approach to apply to your own work?*

Provide opportunity for students to return to their work to revise, modify, and finalize their template.

Once they have finalized their template, consider having students display their template under a document camera or overhead, so that classmates can share what they might “see” in the template, e.g., a flower, a ship, a car, an animal, etc. This may help students with adding details to their template to create their artwork.

Provide the opportunity for students to return to their work to revise, modify, and finalize the design of their template.

Provide additional time for students to complete their artwork and determine a way to tell their story. You may opt to host a gallery walk when all pieces are complete and invite community members to attend.

In Example 2, students are:

- given a cognitively demanding task. They are asked to consider how transformations can be used to create a figure that tessellates a plane, create the template using one or more transformations, and create a tessellation using the template and adding details to bring meaning to the tessellation (LP 1).
- working individually and collaboratively in pairs and sharing their findings with the class as they progress through this exploration. Students are encouraged to ask questions to better understand the thinking process (LP 2).
- applying their understanding of transformations to create a figure that will tessellate and examining patterns that emerge from tessellating a plane (LP 3).
- bringing their identity to the fore as they interact with mathematical figures to create a personal piece of artwork (LP 4).

- given insight into how artists integrate mathematics to create meaningful and artistic designs (LP 6).
- encouraged to use dynamic geometry software to facilitate the exploration and construction of templates and tessellation (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Artistic Expression:** [Express myself artistically](#) (FK.AC.2)

References

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