

M102: Modeling with Linear Functions and Equations

[Illustrative Mathematics–Algebra 1 Unit 2: Linear Equations, Inequalities, and Systems](#)

[Illustrative Mathematics–Algebra 1 Unit 3: Two-variable statistics](#)

Notes for improving overall alignment:

- Consider attending to M101 and M102 concurrently. One way to engage students in recognizing the learning they are doing is to give students time to reflect on their learning in connection with the Content and Practice Expectations of this badge as assignments related to this badge surface, i.e., as opportunities to engage in mathematical modeling arise.
- Educators will need to bring in space to attend to [Content and Practice Expectation 102.i](#). Consider using [this template](#) to offer students space to reflect and document their thinking at key points in time. Spaces where we see an opportunity for this are indicated by a triangle ▲.
- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4)**. This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- While most of the modeling cycle is well represented, there are few opportunities for students to formulate their own questions or to identify the variables of interest in a given situation or data set. The use of [Math Language Routine 5: Co-Craft Questions](#) may be useful in building these opportunities into some of the existing modeling tasks within the unit, particularly some of the more complex tasks toward the end of the unit.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one’s thinking has excellent value in solidifying one’s knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students’ identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental

challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you...
102.a: Engage in the modeling cycle (performance assessment).	i. engage with the full modeling cycle (problem, formulate, compute, interpret, validate, revise as necessary, report).
102.b: Interpret linear functions and equations that arise in applications in terms of the context.	i. select two different data points that illustrate something interesting or important about the context you are modeling and describe what these points tell you about the topic.
	ii. use a model to compute output values and then interpret the values in the context of the problem.
	iii. describe the domain of the linear function, including any noticings about values that aren't part of the domain given the context.
	iv. describe the contextual meaning of the rate of change for a linear model.

Content and Practice Expectations	Indicators Choose an artifact where you...
102.c: Analyze linear functions using different representations.	i. create a linear graph to show a particular context and describe important values that help to understand something about the problem you are exploring.
	ii. explain how the parameters of a linear model relate to its graph.
102.d: Build linear functions that model relationships between two quantities.	i. identify the variables of interest in a given situation or data set that are interesting and that demonstrate a linear relationship.
	ii. build a function that models the linear relationship between two quantities in a situation.
102.e: Analyze and solve linear equations and pairs of simultaneous linear equations to draw conclusions.	i. find the solution to a system of two linear equations in two variables.
	ii. explain whether a system of two linear equations will have one solution, no solution, or an infinite number of solutions.
	iii. create a system of two linear equations in two variables to model a real-world situation.
	iv. explain the meaning of a solution to a system of two linear equations in two variables in terms of the context being modeled.
102.f: Interpret expressions for linear functions in terms of the situation they model.	i. describe the meaning of the parameters for a linear function in terms of the context.
102.g: Solve linear equations and inequalities in one variable.	i. find the solution to a linear equation or inequality in one variable.
	ii. justify a solution to a linear equation or inequality in one variable using logical reasoning or an argument about reasonableness.

Content and Practice Expectations	Indicators Choose an artifact where you...
102.h: Create equations that describe linear relationships.	i. identify the variables of interest in a given situation or data set that are interesting and that demonstrate a linear relationship.
	ii. build an equation that models the linear relationship between two quantities in a situation.
102.i: Understand the relevance of modeling with linear functions.	i. see something that surprised you or that you learned about the topic through linear modeling of data. Share what you found surprising or what you learned.
	ii. highlight something you are most proud of in your learning about linear modeling.
	iii. explain how modeling can be used to answer important questions.
102.j: Use a linear function model to determine values of interest in a real-world problem.	i. determine one or more output values for specific inputs to the function.

Unit Overview

	<u>L1</u>	<u>L2</u>	<u>L3</u>	<u>L4</u>	<u>L5</u>	<u>L6</u>	<u>L7</u>	<u>L8</u>	<u>L9</u>	<u>MT</u>
102.a.i	X				X					x
102.b.i			X		X					
102.b.ii			X		X					
102.b.iii			X	X	X		X			
102.b.iv			X	X	X		X			

102.c.i			X	X			X			
102.c.ii			X	X			X			
102.d.i			X							
102.d.ii			X				X			
102.e.i						X	X			X
102.e.ii						X	X			
102.e.iii						X				X
102.e.iv						X				X
102.f.i	X	X				X				X
102.g.i								X	X	
102.g.ii		X						X	X	
102.h.i	X	X	X							X
102.h.ii	X	X	X							X
102.i.i	▲	▲				▲				X
102.i.ii	▲	▲				▲				X
102.i.iii	▲	▲				▲				X
102.j.i										X

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
2.1	K	102.a.i 102.f.i 102.h.i 102.h.ii 102.i.i 102.i.ii 102.i.iii	LP1, LP3, LP6	Context: Planning a pizza party Math focus: Using equations to represent situations
2.8	K	102.f.i 102.g.ii 102.h.i 102.h.ii 102.i.i 102.i.ii 102.i.iii	LP1, LP3, LP6	Context: Parade clean up, filling/emptying tanks Math focus: Rearrange equations to solve for variable of interest
2.10	M Amplify Activity 10.1 and Activity 10.2. Consider introducing the concept of domain and asking about values that are not shown on the graph. Consider having students choose a point	102.b.i 102.b.ii 102.b.iii 102.b.iv 102.c.i 102.c.ii 102.d.i 102.d.ii 102.h.i	LP3, LP5, LP6	Context: Spending money on games and rides at a carnival, number of nickels and dimes Math focus: Connecting equations with graphs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	on the graph and explain why they chose the point based on the context. Skip Activity 10.3 to give time for exploring suggested ideas.	102.h.ii		
2.11	M Focus on Activity 11.2 and connections between the structure of two-variable linear equations, their graphs, and the situations they represent. To better align to CPEs listed, consider asking students about the domain and prompting students to interpret the rate of change in terms of the contexts. Consider having students choose a point on the graph and explain the meaning of that point in terms of the context. Skip Activity 11.1 and	102.b.iii 102.b.iv 102.c.i 102.c.ii	LP3, LP5, LP6	Context: Gallons of water in a tank, how much money is spent on pounds of almonds and dried figs Math focus: Connecting equations to graphs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	Activity 11.3 to give time for exploring suggested ideas.			
3.4 Add Unit 3 Lesson 4 Activities as described	<p>M - Two days. Please note this lesson is also included in Badge M112. Note that the following is based on using that lesson as an introduction to the messiness mathematical modeling can involve. Mastery of the topics introduced in that lesson are not expected at this stage. Please focus on the CPEs listed to guide discussion.</p> <p>Day 1: Activity 4.1 and Activity 4.2.</p> <p>Day 2: Activity 4.3 and Activity 4.4. For Activity 4.4, consider</p>	<p>102.a.i 102.b.i 102.b.ii 102.b.iii 102.b.iv</p>	LP3, LP6	<p>Context: Oranges on a scale, ingredient cost vs. sales price, age vs. reaction time, number of bananas vs. money, room size vs. cost to install flooring, temperature vs. volume</p> <p>Math focus: Linear models</p>

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	having students create a problem/situation based on one of the graphs and interpret the graph based on the context.			
2.12	<p>M - Two days. Please note this lesson is also included in Badge M101. Consider having students engage in this through a modeling lens as described below, then return to it on Day 2 to attend to the Badge M101 CPEs.</p> <p>Day 1: 12.1-12.2 12.2. Consider focusing on CPE 102.e.ii by having students question how many solutions there could be in the situation.</p> <p>Day 2: 12.3 Assign one graph per</p>	<p>102.e.i 102.e.ii 102.e.iii 102.e.iv 102.f.i 102.i.i 102.i.ii 102.i.iii</p>	LP1, LP3, LP4, LP6, LP7	<p>Context: Trail mix, dining halls, purchasing family tickets to a magic show, buying posters</p> <p>Math focus: System of equations</p>

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	team. Consider focusing on CPE 102.e.iv by having students explain what the solution means in terms of the context.			
2.17	<p>M</p> <p>Please note Activities 17.1, 17.3, and 17.4 from this lesson are best suited for Badge M101.</p> <p>Use Activity 17.2 and utilize recommended Math Language Routines as well as <u>Co-Crafting Questions</u> in the launch of this activity to attend to aspects of mathematical modeling. To better align to CPEs listed, consider asking students about the domain and prompting students to interpret the rate of change in terms of the contexts. Consider having students choose</p>	<p>102.b.iii</p> <p>102.b.iv</p> <p>102.c.i</p> <p>102.c.ii</p> <p>102.d.ii</p> <p>102.e.i</p> <p>102.e.ii</p>	LP2, LP3, LP7	<p>Context: Pool passes and gym memberships</p> <p>Math focus: Number of solutions for systems of equations</p>

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	a point on the graph and explain the meaning of that point in terms of the context.			
2.18	M 18.2: Encourage students to come up with the inequalities based on the context, rather than giving them the inequalities.	102.g.i 102.g.ii	LP1, LP2, LP3, LP5	Context: Planning a senior ball, constraints for an elevator Math focus: Representing situations in linear inequalities
2.19	M For Activity 19.2: Launch the activity by <u>Co-Crafting Questions</u> Have students justify which orchard they would choose based on the inequalities.	102.g.i 102.g.ii	LP1, LP3, LP5, LP6	Context: Class field trip to an orchard, job to pay for tuition Math focus: Solutions to linear inequalities in one-variable

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
Modeling Task- <u>A</u> <u>New Heating System</u>	M Consider ending the modeling task by having students reflect on the modeling process—highlight what they learned, what they are most proud of, and how modeling can be used to answer important questions.	102.a.i 102.e.i 102.e.iii 102.e.iv 102.f.i 102.h.i 102.h.ii 102.i.i 102.ii 102.i.iii 102.j.i	LP1, LP3, LP4, LP5, LP6, LP7	Context: Replacing an old heating system Math focus: Graphing system of equations