

# M201: Function Concepts

[Illustrative Mathematics–Algebra 2 Unit 1: Sequences and Functions and Algebra 2 Unit 5: Transformations of Functions and Algebra 1 Unit 7: Quadratic Equations](#)

## Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4)**. This can be accomplished by learning about students’ interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum’s mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.

## Learning Principles

**Engage with cognitively demanding tasks in heterogeneous settings (LP 1).** Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

**Engage in social activities (LP 2).** Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one’s thinking has excellent value in solidifying one’s knowledge.

**Build conceptual understanding through reasoning (LP 3).** Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

**Have agency in their learning (LP 4).** Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students' identities and interests to the fore and build on the strengths that they bring to the learning space.

**View mathematics as a human endeavor across centuries (LP 5).** Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

**See mathematics as relevant (LP 6).** Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

**Employ technology as a tool for problem-solving and understanding (LP 7).** Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

## Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you...
201.a: Understand the concept of a function and use function notation.	i. demonstrate understanding of the following language of functions: If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .
	ii. recognize that sequences are functions, sometimes defined recursively, and their domain is a subset of the integers.
201.b: <b>Graph</b> functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	i. graph linear and quadratic functions and show intercepts, maxima, and minima.
	ii. graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
	iii. graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
	iv. graph exponential functions showing intercepts and end behavior.
201.c: <b>Write</b> a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	i. rewrite expressions, using processes such as factoring and completing the square in a quadratic function, to show zeros, extreme values, and symmetry of graphs.
	ii. use the properties of exponents to interpret expressions for exponential functions.
201.d: <b>Build</b> new functions from existing functions.	i. identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $k f(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
201.e: <b>Compare</b> properties of two functions each represented in a different	i. compare properties of two functions that highlight two quantities or features of interest.

Content and Practice Expectations	Indicators Choose an artifact where you...
way (algebraically, graphically, numerically in tables, or by verbal descriptions).	
201.f: <b>Build</b> a function that models a relationship between two quantities.	i. identify and/or define the variables of interest in a given situation or data set that demonstrate a functional relationship.
	ii. build a function that models the relationship between two quantities in a situation.

## Unit Overview—Algebra 2 Unit 1: Sequences and Functions

	<a href="#">L1</a>	<a href="#">L2</a>	<a href="#">L3</a>	<a href="#">L4</a>	<a href="#">L5</a>	<a href="#">L6</a>	<a href="#">L7</a>	<a href="#">L8</a>	<a href="#">L9</a>	<a href="#">L10</a>	<a href="#">L11</a>	<a href="#">MT1</a>
201.a.i			X		X	X	X	X	X	X	X	
201.a.ii	X	X	X		X	X	X	X	X	X	X	X
201.b.i			X				X					
201.b.ii												
201.b.iii												
201.b.iv		X				X	X					

201.c.i												
201.c.ii								X	X			
201.d.i												
201.e.i						X		X		X		
201.f.i												
201.f.ii												

Note: Columns in gray indicate a recommendation to delete the lesson

## Unit Overview—Algebra 2 Unit 5: Transformations of Functions and Algebra 1 Unit 7: Quadratic Equations

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	MT2	L12	L13	L14	L15	L16	L17	MT3
201.a.i	X	X	X	X		X	X	X	X	X	X		X	X				X	X
201.a.ii																			
201.b.i	X							X		X	X	X	X			X		X	X
201.b.ii		X	X						X	X	X	X							
201.b.iii	X	X		X		X		X		X									
201.b.iv	X		X					X		X	X								
201.c.i														X	X	X	X	X	X

201.c.ii																			
201.d.i	X	X	X	X	X	X	X	X	X		X	X							X
201.e.i			X	X	X				X										
201.f.i												X							
201.f.ii		X	X				X			X	X	X							X

Note: Columns in gray indicate a recommendation to delete the lesson

## Guidance for Lesson Priority—Algebra 2 Unit 1: Sequences and Functions

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
1.1 Unit 1	K	201.a.ii	LP1, LP2, LP3, LP6	<b>Context:</b> The Tower of Hanoi  <b>Math focus:</b> Giving examples of sequences (described recursively)
1.2	K	201.a.ii 201.b.iv	LP2, LP3, LP6	<b>Context:</b> Paper slicing  <b>Math focus:</b> Finding missing terms in a geometric sequence
1.3	K	201.a.i 201.a.ii 201.b.i	LP2, LP3	<b>Context:</b> Purely mathematical  <b>Math focus:</b> Explaining what it means for a sequence to be arithmetic or geometric
1.4	D Lesson listed as optional by authors. The focus is on learning to use spreadsheets and graphing technology without context. Recommend including use of tech tools, like graphing calculators and			<b>Context:</b> Purely mathematical—use of spreadsheets  <b>Math focus:</b> Using technology to graph a sequence and create many terms of a sequence

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	spreadsheets, within other lessons so that LP7 is integrated into learning experiences.			
1.5	K	201.a.i 201.a.ii 201.f.ii	LP2, LP3	<b>Context:</b> Visual pattern of dots  <b>Math focus:</b> Defining arithmetic and geometric sequences recursively using function notation
1.6	K Lesson is listed as optional by the authors. Yet, this lesson provides more practice writing recursive definitions.	201.a.i 201.a.ii 201.b.iv 201.e.i 201.f.ii	LP2, LP3	<b>Context:</b> Purely mathematical—visual patterns  <b>Math focus:</b> Representing sequences in different ways
1.7	K	201.a.i 201.a.ii 201.b.i 201.b.iv 201.f.ii	LP2, LP3, LP5	<b>Context:</b> Purely mathematical  <b>Math focus:</b> Asking questions to get information to represent a sequence

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
1.8	K	201.a.i 201.a.ii 201.c.ii 201.e.i 201.f.i 201.f.ii	LP2, LP3, LP5	<b>Context:</b> Paper slicing; a Sierpinski triangle  <b>Math focus:</b> Explaining why different equations can represent the same sequence
1.9	K	201.a.i 201.a.ii 201.c.ii 201.f.i 201.f.ii	LP2, LP3, LP6	<b>Context:</b> Taking cake; Fibonacci squares  <b>Math focus:</b> Representing situations with sequences
1.10	K	201.a.i 201.a.ii 201.e.i 201.f.i 201.f.ii	LP2, LP3, LP6, LP7	<b>Context:</b> Population; square patterns  <b>Math focus:</b> Defining sequences using equations
1.11	K	201.a.i 201.a.ii 201.f.i 201.f.ii	LP2, LP3, LP6	<b>Context:</b> Paper trail; Koch snowflake  <b>Math focus:</b> Determining the sum of a sequence representing a situation

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
Modeling Task 1- <a href="#">Viral Marketing</a>	Three versions are given, with a varied amount of information given to students.	201.a.ii 201.f.i 201.f.ii	LP1, LP3, LP4, LP6, LP7	<b>Context:</b> Marketing campaign  <b>Math focus:</b> Modeling different scenarios with exponential functions

## Guidance for Lesson Priority—Algebra 2 Unit 5: Transformations of Functions and Algebra 1 Unit 7: Quadratic Equations

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.1	K	201.a.i 201.b.i 201.b.iii 201.b.iv 201.d.i	LP2, LP3, LP6	<b>Context:</b> Soda water temperature; describing and sketching transformed graphs  <b>Math focus:</b> Describing how a graph is transformed

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.2	K	201.a.i 201.b.ii 201.b.iii 201.d.i 201.f.ii	LP3, LP6	<b>Context:</b> A bakery kitchen's temperature  <b>Math focus:</b> Using function notation to represent a vertical or horizontal translation
5.3	K	201.a.i 201.b.ii 201.b.iv 201.d.i 201.e.i 201.f.ii	LP3, LP6	<b>Context:</b> Bakery kitchen; thawing meat  <b>Math focus:</b> Writing equations to represent vertical and horizontal translations
5.4	K	201.a.i 201.b.iii 201.d.i 201.e.i	LP2, LP3	<b>Context:</b> Purely mathematical  <b>Math focus:</b> Reflecting a graph across the x- and y-axis
5.5	K	201.d.i 201.e.i	LP1, LP2, LP3	<b>Context:</b> Ferris wheel  <b>Math focus:</b> Identifying even and odd functions by their graphs

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.6	K	201.a.i 201.b.iii 201.d.i	LP2, LP3	<b>Context:</b> Purely mathematical  <b>Math focus:</b> Completing even and odd functions given half the graph; identifying even and odd functions by their equations
5.7	K	201.a.i 201.b.iv 201.d.i 201.f.ii	LP3, LP6, LP7	<b>Context:</b> Temperature of soda water left out  <b>Math focus:</b> Writing an equation from a description of how a graph is transformed
5.8	K	201.a.i 201.b.i 201.b.iii 201.d.i	LP3, LP6, LP7	<b>Context:</b> Hulme Arch Bridge; feeding the dog  <b>Math focus:</b> Calculating the scale factor needed to transform the output of a function to model data
5.9	K	201.a.i 201.b.ii 201.d.i 201.e.i	LP2, LP3, LP6	<b>Context:</b> Distance walked and Ferris wheel  <b>Math focus:</b> Describing the effect of a scale factor on the input of a function; understanding the differences between scaling the outputs and scaling the inputs of a function

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.10	K Students are combining functions given in graphical form to deepen conceptual underpinnings of CPE.201.b.	201.a.i 201.b.i 201.b.ii 201.b.iii 201.b.iv 201.f.ii	LP2, LP3, LP6	<b>Context:</b> Book sales  <b>Math focus:</b> Combining two functions in different ways
5.11	K	201.a.i 201.b.i 201.b.ii 201.b.iv 201.d.i 201.f.ii	LP1, LP2, LP3, LP6, LP7	<b>Context:</b> Temperature of bottled water  <b>Math focus:</b> Transforming a function so its graph models a data set
Modeling Task 2- <a href="#">Swing Time</a>	Two versions are given, with more information given in the second.	201.b.i 201.b.ii 201.d.i 201.f.i 201.f.ii	LP1, LP2, LP3, LP5, LP6, LP7	<b>Context:</b> Period of a pendulum  <b>Math focus:</b> Building a best-fit function using transformations

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
7.2  Algebra 1 Unit 7	M Replace Activity 2.1 with warm-up from <u>A1 Unit 7 Lesson 1: What goes up must come down</u> to launch this lesson.	201.a.i 201.b.i	LP3, LP6, LP7	<b>Context:</b> Potato launch; ticket sales revenues  <b>Math focus:</b> Connect factored and standard form of quadratic functions and see solution in graph of the function
7.3 and 7.4	M Merge these two lessons as recommended below: Activity 3.1, Activity 3.2, Activity 3.2: Are you ready for more?, and Activity 4.3.	201.a.i 201.c.i	LP1, LP3, LP6, LP7	<b>Context:</b> Projectile motion  <b>Math focus:</b> Zero product property
7.5	M Foundation for CPE 201.c.i. Amplify Activities 5.2 and 5.4.	201.c.i	LP1, LP3	<b>Context:</b> Purely mathematical  <b>Math focus:</b> Graphing and rewriting quadratic equations in the form of expression = 0
7.6, 7.7, and 7.9	M Merge these lessons as recommended below: Activity 6.2, Activity 7.2, Activity 7.3, and Activity	201.b.i 201.c.i	LP1, LP3, LP7	<b>Context:</b> Purely mathematical  <b>Math focus:</b> Rewriting quadratic expressions in factored form

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	9.3.			
7.11, 7.12, and 7.14	M Merge these lessons as recommended below: Activity 11.2, Activity 12.2, Activity 12.3, and Activity 14.3.	201.c.i	LP1, LP3, LP7	<b>Context:</b> Purely mathematical  <b>Math focus:</b> Perfect squares and completing the square
7.23	K	201.b.i 201.c.i	LP1, LP3, LP7	<b>Context:</b> Ticket sales  <b>Math focus:</b> Determining maximum and minimum values of a quadratic function
Modeling Task 3- <a href="#">Planning a Concert</a>	K	201.a.i 201.b.i 201.c.i 201.d.i 201.f.ii	LP1, LP2, LP3, LP5, LP6, LP7	<b>Context:</b> Optimizing profit for a concert  <b>Math focus:</b> Using a quadratic function to determine optimal price for a concert