

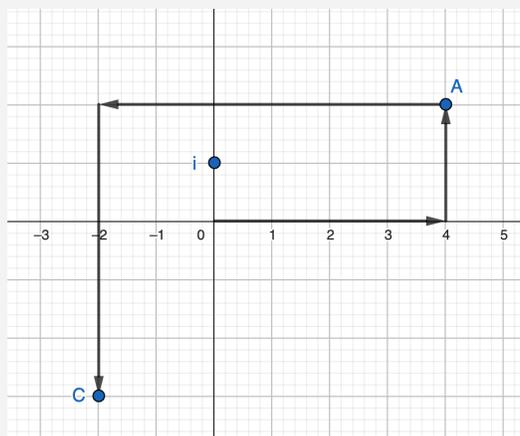
# Annotated Examples: M202 Rational Exponents and Complex Numbers

The examples that follow are intended to illustrate how the learning principles are used to support students' engagement with the content and practices outlined in this badge. These examples do not provide comprehensive coverage of those expectations, but rather elevate some of the learning principles that are less likely to be part of published curricular materials for mathematics instruction. The examples that follow were developed by the Math Badging writing team, unless otherwise specified. These are a small sample of types of learning activities that can be done with students, both in and out of a traditional classroom setting.

## Example 1: Exploring Complex Numbers

Students are presented with a problem like this:

1. The diagram represents the sum of two complex numbers:  $(4 + 2i) + (-6 - 5i)$ .



- a. Which part of the expression does Point A represent? Explain how you know.
  - b. What complex number does Point C represent? Explain how you know.
  - c. Based on the analysis above, describe how to find the sum of two complex numbers.
2. Write and record a tutorial for a new classmate that explains how to simplify expressions that involve addition or subtraction of two complex numbers. Be sure to include a diagram that illustrates why your method works.

### Sample Learning Activity

Provide time for students to consider Question 1 and independently develop a response. Engage students using the Math Language Routine 1: Stronger and Clearer Each Time (MLR 1), as described below:

1. Have students write their response to 1c, including their justification.
2. If you have emerging multilingual students in your class, give students a minute to visualize themselves verbalizing their thoughts to a classmate.
3. Ask students to meet with a classmate and take turns to share what they wrote (about two minutes each). As one person shares, the other should pose clarifying questions.
4. Give students about one minute to reflect on this partner sharing. Encourage students to add new thinking to their original response based on what they heard.
5. Have students switch partners and repeat the process of sharing their thinking and refining their response. Repeat this process as much as time permits.

Provide time for students to revise their written justifications using the information gained during their partner-sharing time. While observing student discussions, identify varying solution methods to be presented during the whole group share out. Look for methods that invite connections between the graphical representation and a purely arithmetic approach. Have selected students present their responses—consider using a document camera for students to show actual work—and explain their justification. Focus the discussion on Question 1c. As discussion takes place, invite classmates to join the discussion. Ask:

- *How do you differentiate the real and imaginary axis?*
- *Why does each axis have a positive and negative side?*
- *Do you agree with \_\_\_\_ explanation of Point A? What about this explanation is clear to you? How can we strengthen the explanation?*
- *Is there anything you disagree with in this explanation??*

Provide time for students to revise their work.

Use student responses to Question 1c to launch work on Question 2. Pose a question like, “Will the method you described (1c) work for any pair of complex numbers?” Encourage students to explore concrete examples as they prepare their response to Question 2.

Consider inviting select students to share their approach at strategic points in time as they work on Question 2. As students present their ideas, invite classmates to reflect using the following questions:

- *What makes sense about the ideas shared?*
- *What questions surface for you?*
- *What can you take from this discussion and apply to your own work?*

Provide time for students to revise their work. As students continue their work, select additional pairs to share how they structured their investigation to determine a response to the question, “Does this method work for any two complex numbers?” Give students additional time to revise their work.

In this example, students are:

- engaged in a cognitively demanding task where students use their understanding of the Cartesian plane to make sense of the complex plane. Students then leverage the structure of the plane to build an explanation for the arithmetic of complex numbers (LP 1).
- given the opportunity to learn and build from each other’s explanations for the arithmetic of complex numbers (LP 2).

- given the opportunity to build conceptual understanding of the underlying structure of the complex number system as they reason and work to explain how to add and subtract complex numbers. In building their mathematical argument, students have the opportunity to explore multiple cases of  $(a \pm bi) + (c + di)$  (LP 3).
- given the opportunity to develop their mathematical identity since their ideas and reasoning are the foundation of the discussion (LP 4 and LP 5).
- encouraged to use technology to create representations of the cases they examine and the argument they work to build (LP 7).

### Example 2: Exploring Rational Numbers

Students are presented with a problem like this:

#### Part 1

Let's think about what  $a^{\frac{1}{2}}$  means.

1. Select some value for  $a$  and develop a table to show some values for  $a^x$ .
2. Based on the values from your table, make an educated guess about the value of  $a^{\frac{1}{2}}$ . Explain your reasoning.
3. Check the actual solution for  $a^{\frac{1}{2}}$ . Explain why this answer makes sense. If the answer is different from your guess above, write down two to three ideas or questions you can pose to help examine why  $a^{\frac{1}{2}}$  has the designated value.
4. Based on conclusions you've reached, what do you anticipate something like  $a^{\frac{3}{2}}$  means?
  - a. How can you use what you see in your table or what you know about properties of exponents to explain what this means?

#### Part 2

5. Based on the discussion of Part 1, what do you anticipate the values of  $a^{\frac{1}{3}}$  and  $a^{\frac{1}{4}}$  will be?
  - a. How can you use what you see in your table or what you know about properties of exponents to explain what this means?
6. Create a video or animated short explaining what an expression like  $d^{\frac{m}{p}}$  means.

### Sample Learning Activity

#### Part 1

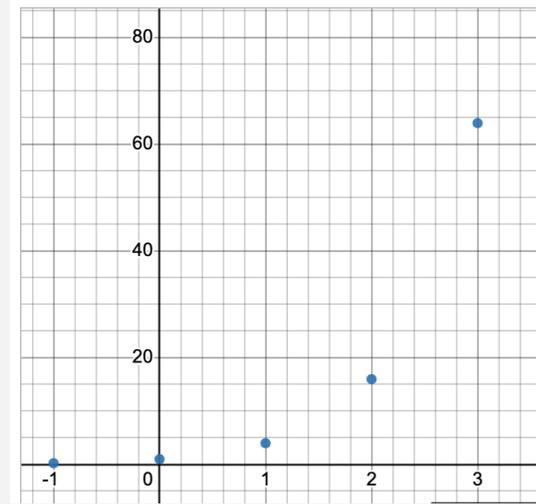
After some individual work time, where students develop an approach to the prompt, encourage them to share and compare approaches. Students should have access to tech tools that facilitate problem solving, like spreadsheets and scientific and graphing calculators. As students share with partners, monitor and select a couple of approaches to highlight and reflect on progress made at this point. Look for student work that offers space to discuss choices students made about values to use as the base

and exponents, as well as a variety of ways that students reasoned about this. Some sample approaches are illustrated below.

$4^0$	$4^{\frac{1}{2}}$	$4^1$	$4^2$	$4^3$
		4	16	64

$2^0$	$2^{\frac{1}{2}}$	$2^1$	$2^2$	$2^3$
		2	4	8

$25^0$	$25^{\frac{1}{2}}$	$25^1$	$25^2$	$25^3$
		25	625	15625



Questions that can be posed to student pairs as they work and/or as students share with classmates:

- *How did you decide on a value of  $a$  to use?*
- *How are you selecting powers of  $a$  to use? Which powers of  $a$  are helpful in reaching a conclusion?*
- *Did you notice anything interesting about the values? What is happening to your numbers?*
- *How did you reach a conclusion about the meaning of a fraction as an exponent?*

Give selected pairs of students time to share responses and invite classmates to reflect on the following questions:

- *What's promising about this approach?*
- *What questions does this approach surface for you?*
- *What can you take from this approach to apply to your own work?*

Provide additional time for students to revise their work, as well as to make progress on the additional prompts in Part 1. While they work, select additional pairs to share how they used the structure of the expressions and their understanding of the values, either by way of reasoning through the values in the table and/or graphs, to reach conclusions. Give students additional time to revise their work. Consider the benefits of holding whole class discussions after Question 2 and again after Question 4. Chart key findings that students can build from to transition to Part 2.

## Part 2

After some individual work time, where students develop an approach to the prompt, encourage them to share and compare approaches. Consider using similar prompts as listed above to give space for students to develop a strategic approach. After a whole group discussion of Question 5, provide

additional time for students to revise their work and to develop their response to Question 6. Showcase student videos and animations. Give students an opportunity to reflect on this experience and revise their work as they see fit. Some questions for reflection are as follows:

- *What is something you learned about yourself in conducting this mathematical investigation?*
- *What is something you're proud of?*
- *What is a new insight you have about how mathematics works?*

In this example, students are:

- engaged in a cognitively demanding task. They are asked to develop a mathematical investigation where they have an opportunity to analyze the pattern of whole number exponents to make sense of fractional exponents. Students have multiple entry points to reason about these expressions in various ways (LP 1).
- working collaboratively in pairs to complete the task, allowing for social interaction with peers. Students are also encouraged to ask their peers questions to understand each other's thinking process and revise their own thinking (LP 2).
- attending to a meaning of exponents as being repeated multiplication and working to make sense of the relationships between whole and fractional exponents. In so doing, students learn to utilize various math tools to reason about and define rational exponents (LP 3).
- given space to develop skills of expert learners, thereby developing agency, as they reflect on their approach to the prompt and incorporate new ways of thinking about this from their peers (LP 4).
- given the opportunity to develop their mathematical identity as they are in the driver's seat throughout this investigation (LP 5).
- using technology as a tool for testing conjectures they develop along the way (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Making Meaning:** [Synthesizing information](#) (FL.ID.1.a)

## References

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