

Annotated Examples: M113 Modeling with Probability

The examples that follow are intended to illustrate how the learning principles are used to support students' engagement with the content and practices outlined in this badge. These examples do not provide comprehensive coverage of those expectations, but rather elevate some of the learning principles that are less likely to be part of published curricular materials for mathematics instruction. The examples that follow were developed by the Math Badging writing team, unless otherwise specified. These are a small sample of types of learning activities that can be done with students, both in and out of a traditional classroom setting.

Example 1: The Titanic

Students are given a task like this:

Part 1: The Titanic

On April 15, 1912, the Titanic struck an iceberg and rapidly sank with only 710 of her 2,204 passengers and crew surviving. Some believe that the rescue procedures favored the wealthier first class passengers. Others believe that the survival rates can be explained by the "women and children first" policy. Data on survival of passengers are summarized in the table below. Investigate what might and might not be concluded from the given data.

	Survived	Did not survive	Total
Children in first class	4	1	5
Women in first class	139	4	143
Men in first class	58	118	176
Children in second class	22	0	22
Women in second class	83	12	95
Men in second class	13	154	167
Children in third class	30	50	80
Women in third class	91	88	179
Men in third class	60	390	450
Total passengers	500	817	1317

Part 2: Extending the Learning

1. *What additional questions surface for you as a result of the work above?*
2. *What are your thoughts and feelings regarding what you have learned about travel in 1912?*
3. *Are there any current events you believe to have similar disparities as those seen in the survival rates of the Titanic? Work to formulate a plan for gathering data about this event and conduct an investigation to see what might or might not be concluded from that data.*
4. *Based on your analysis, what conclusions have you reached? What would you like to see happen?*

Sample Learning Activity

Part 1: The Titanic

Launch this task by displaying the following information about the Titanic and passenger survival data. Give students time to study, and pose the questions: What do you notice? What do you wonder?

On April 15, 1912, the Titanic struck an iceberg and rapidly sank with only 710 of her 2,204 passengers and crew surviving. Data on survival of passengers are summarized in the table below.

	Survived	Did not survive	Total
First class passengers	201	123	324
Second class passengers	118	166	284
Third class passengers	181	528	709
Total passengers	500	817	1317

After some quiet time, give students an opportunity to share their ideas with a partner. As students share, listen for observations and questions that get at how class and survival rate might be related. Capture ideas that surface on a visible surface, like chart paper or white board. Leverage this part of the conversation to then transition to the prompt presented. Refer to student generated observations and questions as applicable.

As students work, notice various ways students make sense of the information and formulate a plan for responding to the prompt. Select some students or partner pairs to share their thinking with the class. Consider using questions below to facilitate discussion:

- *What's promising about this approach?*
- *What questions does this approach surface for you?*
- *What can you take from this approach to apply to your own work?*

Following the discussion, give students an opportunity to reflect on and revise their work. Use questions students generate to further engage students with this data, as well as to amplify the mathematics in play.

Part 2: Extending the Learning

Give students time to work individually or with a small group to identify current events they'd like to investigate in this manner. Monitor and select student voices to share their preliminary progress, focusing the discussions on the formulation of their plans. Consider using the questions below to facilitate discussion:

- *What's promising about this approach?*
- *What questions does this approach surface for you?*
- *What can you take from this approach to apply to your own work?*

Following the discussion, give students an opportunity to reflect on and revise their work.

Give additional time for students to conduct their investigation and present their findings to the class or other audience they feel is appropriate.

In this example, students are:

- given a cognitively demanding task. It requires that students formulate a plan to answer the question at hand. Students also have the opportunity to apply the knowledge and skills from investigating the survival rates of the Titanic to a current event of their choice (LP 1).
- set up to have social interactions, as they share and discuss their work with each other. These interactions may happen live in a physical classroom or asynchronously in a remote setting (LP 2).
- deepening their understanding of conditional probability and independence as they work to make sense of data in two-way tables as well as when they work to gather data for Part 2 (LP 3).
- able to explore skills and dispositions around what it means to model with probability. As they engage in this work, students have the opportunity to develop identities and interests in using mathematics to make sense of the world (LP 4).
- developing a sense of belonging as they formulate and carry out an approach to the challenges presented. This offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for (LP 5).
- given insight into how statisticians work to interpret data and determine associated probabilities of events (LP 6).
- using technology as a tool, as they employ spreadsheet and/or graphing software to perform their analysis (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Diverse Perspectives:** [Holders of power](#) (GC.SA.2.a)

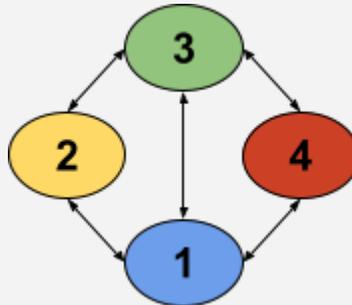
Example 2: Random Walks

Students are given a task like this:

In this task you will explore the connection between random walks and internet searches.

Part 1: Random Walks

Let's begin by exploring how random walks work. Consider the following map with four points of interest.

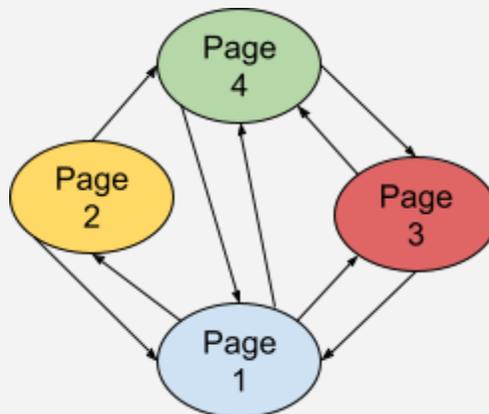


1. *If someone starts at one of the points and randomly decides where to go next, what do you anticipate will happen? Make at least three predictions.*
2. *Design and carry out a simulation to test one of your predictions. Determine ways to quantify how often someone can end up at one of the points.*
3. *What did you learn from designing a simulation to test your predictions?*
4. *What do you anticipate the random walk phenomenon has to do with internet searches?*

Part 2: Internet Searches

Watch this short video [The Internet: How Search Works](#) (Code.org., 2017)

1. *What connections do you see between the work on Part 1: Random Walks and internet searches?*
2. *Let's examine how internet pages are ranked by building on what you've learned about random walks. To simplify things, let's consider an internet with four pages that have links to each other as illustrated in the diagram below.*
 - a. *What do you notice about the relationships between the pages? How do you anticipate the pages will rank?*



3. *Design and carry out a simulation to test your prediction. Based on the simulation, how do the*

pages rank? Does this surprise you?

- a. What is the relative frequency of visits to each page? What do these percentages mean?
4. In the video, we hear Akshaya explain how “spammers try to game the search algorithm.” How might you modify the map above to illustrate how this works? Design a simulation as before and prepare to share your results.
5. Based on your current understanding, what might explain why people have different experiences when they search the internet? What are your thoughts on this?

(adapted from Dr. Claudio Gómez-González, Carleton College)

Sample Learning Activity

Part 1: Random Walks

Introduce the concept of random walks by displaying the map in Part 1: Random Walks. Ask students: what do you notice? What do you wonder? Record student responses on a whiteboard or chart paper to refer to as appropriate.

Ask students to consider what would happen if someone were to start at oval 1 and use something like a die or coin to decide which way to go next. Share with students that this process of randomly selecting which way to move is called a *random walk* in mathematics and has several applications to things like the physical phenomena called [Brownian motion](#) and searching the internet.

In this task, students will explore the connection between random walks and the process of searching the internet. Consider providing a 12-sided die and/or use of technology for generating random numbers.

Next, give students time to engage in prompts individually and in small groups. As students work, monitor for various ways students formulate a plan to respond to the prompt. Make note of how students attend to the various aspects of designing a simulation: number of trials, method for recording, and assignment of outcomes of a die, or how to use technology to determine the movement from node to node. Select some students or partner pairs to share their preliminary plans with the class. Consider using the questions below to facilitate discussion:

- *What’s promising about this approach?*
- *What questions does this approach surface for you?*
- *What can you take from this approach to apply to your own work?*

Following the discussion, give students an opportunity to reflect on and revise their work. Give additional time for students to conduct their investigation and present their findings to the class.

Finally, share one or more reasonable models and give students an opportunity to reflect on the reasonable models and revise their work. Use student responses to Question 4 to launch Part 2: Internet Searches.

Part 2: Internet Searches

Share the video and give students time to discuss Questions 1 and 2 with group mates. Record connections and questions students share and use them throughout the discussion as appropriate. Amplify ideas that get at the connection between random walks and internet searches, such as the idea that we can devise a simulation to understand how ranking works. This is the focus of Part 2.

Next, give students time to engage in Question 3 individually and in small groups. As students work, monitor for various ways students formulate a plan to respond to the prompt. Make note of how students attend to the various aspects of designing a simulation: number of trials, method for recording, and assignment of outcomes of a die, or how they use technology, to determine the movement from node to node. Select some students or partner pairs to share their preliminary plans with the class. Consider using the questions below to facilitate discussion:

- *What's promising about this approach?*
- *What questions does this approach surface for you?*
- *What can you take from this approach to apply to your own work?*

Following the discussion, give students an opportunity to reflect on and revise their work. Give additional time for students to conduct their investigation and present their findings to the class. Repeat the process above for Question 4.

Finally, share one or more reasonable models and give students an opportunity to reflect on the reasonable models and revise their work. Consider having students design a public service announcement on the ideas they have shared in response to Question 5.

In this example, students are:

- given a cognitively demanding task. It requires that students formulate a plan to answer the question at hand. Students also have the opportunity to apply the knowledge and skills from making sense of random walks to make sense of how search engines work (LP 1).
- set up to have social interactions, as they share and discuss their work with each other. These interactions may happen live in a physical classroom or asynchronously in a remote setting (LP 2).
- deepening their understanding of probability and independence as they design and carry out a simulation. Students also work to interpret the results of their simulations in terms of the context (LP 3).
- able to explore skills and dispositions around what it means to model with probability. As they engage in this work, students have the opportunity to develop identities and interests in using mathematics to make sense of the world (LP 4).
- developing a sense of belonging as they formulate and carry out an approach to the challenges presented. This offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for (LP 5).
- given insight into how mathematicians use probability to model phenomena like internet searching (LP 6).
- using technology as a tool, as they employ spreadsheet and/or graphing software to perform their simulation and analysis (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Interpreting Data:** [Data tools](#) (FL.MST.3.a)

References

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