

M151: Modeling with Geometry

[Illustrative Mathematics—Geometry Unit 5: Solid Geometry](#)

Notes for improving overall alignment:

- Opportunities for students to engage in pair or group work are an integral part of this curricular design, and special considerations will need to be taken into account if working outside of a traditional classroom setting to give students opportunities to **engage in social activities (LP2)**.
- Leverage the structure of the curriculum to give space for students to **have agency in their learning (LP4)**. This can be accomplished by learning about students' interests and lived experiences, then tailoring curricular prompts to reflect those experiences and interests. Use the curriculum's mathematical learning arcs to structure opportunities and check for progress toward attainment of *Content and Practice Expectations*.
- Leverage the structure of the curriculum and opportunities to use the embedded math language routines to give students space to **view mathematics as a human endeavor across centuries (LP5)**. This can be done by positioning students as authors of important mathematical ideas, where their thinking is at the center of discussion and exploration, as well as by giving time to reflect on progress and personal growth.
- While most of the modeling cycle is well represented, there are few opportunities for students to formulate their own questions or to identify the variables of interest in a given situation or data set. The use of Math Language Routine 5: Co-Craft Questions may be useful in building these opportunities into some of the existing modeling tasks within the unit.
- Educators will need to bring in space to attend to Content and Practice Expectation 151.c.i. Consider using this modeling prompt to offer students an opportunity to engage in this CPE. Additional CPEs aligned with this task are marked by a triangle (▲).

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one’s thinking has excellent value in solidifying one’s knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students’ identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental

challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Content and Practice Expectations

Content and Practice Expectations	Indicators Choose an artifact where you...
151.a: Engage in the modeling cycle (performance assessment).	i. engage with the full modeling cycle (problem, formulate, compute, interpret, validate, revise as necessary, report).
151.b: Use geometric shapes, their measures, and their properties to describe objects.	i. represent a context, situation, or object from outside mathematics as a mathematical object.
	ii. explain how the mathematical object(s) represent the real-life context, situation, or object.
	iii. use properties of the mathematical object to determine quantities of interest and interpret those values within the context.
151.c: Apply concepts of density based on area and volume in modeling situations.	i. apply concepts of density based on area in modeling situations.
	ii. apply concepts of density based on volume in modeling situations.
151.d: Apply geometric methods to solve design problems.	i. design an object or structure to satisfy physical constraints or features in a real-life context.

Content and Practice Expectations	Indicators Choose an artifact where you...
151.e: Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	i. use informal justifications to explain or reason about circumference, area, and volume formulas. Justifications may reflect dissection arguments, Cavalieri's principle, and/or informal limit arguments.
151.f: Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	i. solve real-life and mathematical problems using volume formulas for cylinders, pyramids, cones, and spheres.
151.g: Solve real-world problems involving area, volume, and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	i. analyze the context of the problem to determine which measurable geometric attributes are important to reasoning about the problem.
	ii. solve real-life problems using area, volume, and surface area of two- and/or three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and/or right prisms.
151.h: Use the Pythagorean theorem to solve right triangles in applied problems.	i. recognize situations where the Pythagorean theorem is a useful tool for representing and solving problems.
	ii. solve applied problems using the Pythagorean theorem.

Unit Overview

	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10	L11	L12	L13	L14	L15	L16	L17	L18	MT	MT2
151.a.i																				▲
151.b.i	X	X	X		X		X	X	X	X	X			X		X		X	X	

151.b.ii	X	X	X		X		X	X	X	X	X			X		X		X	X	
151.b.iii			X		X		X	X		X	X		X	X	X	X		X	X	
151.c.i																				▲
151.c.ii																X				
151.d.i	X	X	X				X	X						X	X	X			X	
151.e.i	X	X	X	X		X	X		X	X		X	X							
151.f.i									X	X	X		X	X	X	X	X		X	
151.g.i					X		X	X	X	X				X		X	X	X	X	▲
151.g.ii					X		X	X			X			X		X	X	X	X	▲
151.h.i											X				X					
151.h.ii											X				X					

Note: Columns in gray indicate a recommendation to delete the lesson

Guidance for Lesson Priority

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
5.1	<p>M</p> <p>As students engage in creating and examining 3D images, consider asking students to identify objects they see or have seen in daily life that can be represented using these images to fully attend to expectations of CPE 151.b. Image C in the warm-up gives students an opportunity to learn about “<u>tavillons</u>” as well as explore the reasons why someone would design a building that has a conical rooftop. Consider engaging students in Activity 1.2: “Are you ready for more?”, as well as inviting students to create an image in the</p>	<p>151.b.i 151.b.ii 151.d.i 151.e.i</p>	<p>LP2, LP6, LP5, LP7</p>	<p>Context: Purely mathematical—digital and hand drawn three-dimensional figures</p> <p>Math focus: Axis of rotation, solid of rotation, cylinders, cone, sphere</p>

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
	same fashion of an object they see in their daily life.			
5.2	M Consider asking students to name things they encounter that Activity 2.1: “Slice this”, may apply in order to deepen engagement with CPE 151.b. Utilize Activity 2.3’s “Are you ready for more?” prompt to get to the full extent of CPE 151.b.	151.b.i, ii 151.d.i 151.e.i	LP1, LP2, LP3, LP6, LP7	Context: Coins, three-dimensional slabs, 3D-printer, cheese, toy rocket Math focus: Cross sections, sphere, prism, faces, cylinder, pyramid, cone
5.3	K	151.b.i, ii, iii 151.d.i 151.e.i	LP2, LP3, LP5, LP6	Context: Purely mathematical—pyramid mobile Math focus: Dilation, center of dilation, cross-sections, scale factor
5.4	K Good foundation for engaging in applications grounded in density	151.e.i	LP1, LP2, LP3	Context: Purely mathematical—dilated rectangle, dilated blob Math focus: Dilating by scale factors for

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	(CPE.151.c.i).			area
5.5	K	151.b.i, ii, iii 151.g.i, ii	LP1, LP2, LP6, LP7	Context: Transamerica building, scaling a painting, train station Math focus: Scaling, area, cross-sections, pyramid, rate of change, graphing
5.6	K Good foundation for engaging in applications grounded in density (CPE.151.c.ii).	151.e.i	LP1, LP2, LP3, LP5, LP7	Context: Sierpinski's triangle Math focus: Scaling surface area and volume of cube
5.7	M Consider amplifying Activity 7.3 to give students time to research and make sense of space satellite design. This will amplify connections to LPs listed.	151.b.i, ii, iii 151.d.i 151.e.i 151.g.i, ii	LP1, LP2, LP3, LP5, LP6, LP7	Context: Shipping company, satellite, superhero action figure Math focus: Scale, dilation, graphing, equations, surface area, volume, cube root

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5.8	M If limited on time, take out Activity 8.2.	151.b.i, ii, iii 151.d.i 151.g.i, ii	LP1, LP2, LP3, LP5, LP6	Context: Beverage company, Burj Khalifa, Eiffel Tower, dog food, aquarium Math focus: Dilated solids, scale factors, cylinders
5.9	K This lesson builds on Lesson 5.1.	151.b.i, ii 151.e.i 151.f.i 151.g.i	LP1, LP2, LP3	Context: Water transfer Math focus: Volume of cylinders and prisms, composite figures, hollow cylinder, solids of rotation
5.10	K	151.b.i, ii, iii 151.e.i 151.f.i 151.g.i	LP1, LP2, LP3, LP6, LP7	Context: Stack of coins, index cards Math focus: Right and oblique cylinders, volume, cross-sections, Cavalieri's principle
5.11	K If students complete this badge prior to M155: Right Triangle Trigonometry, then an additional measurement will need to be given for students to be able to use Pythagorean	151.b.i, ii, iii 151.f.i 151.g.i, ii 151.h.i, ii	LP1, LP2, LP3, LP4, LP6	Context: Cayan Tower Math focus: Pythagorean theorem, trigonometric ratio, volume

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	theorem to solve for unknown lengths in various prompts.			
5.12	K	151.e.i	LP1, LP2, LP3, LP4	Context: Purely mathematical—pyramids, cones, cylinders, prisms Math focus: Platonic solids, faces, apex, cross-sections, right and oblique pyramids and other solids
5.13	K This lesson uses the pyramids made in 5.12.	151.b.iii 151.e.i 151.f.i	LP1, LP2, LP3	Context: Purely mathematical—using cross-sections to compare volume of prisms, cylinders, and cones Math focus: Formula for the volume of pyramids
5.14	M Consider amplifying Activity 14.2’s “Are you ready for more?” prompt in lieu of the prompts in Activity 14.2’s “Practice with Pyramids” to engage students with	151.b.i, ii, iii 151.d.i 151.f.i 151.g.i, ii	LP1, LP2, LP4, LP6, LP7	Context: Pyramid of Giza, ice sculpture Math focus: Comparing volume of pyramids, prisms and cones with a focus on pyramids, finding dimensions given volume

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	CPE 151.g.			
5.15	K	151.b.iii 151.d.i 151.f.i 151.h.i, ii	LP1, LP3, LP7	Context: Purely mathematical—strategies for selecting formulas and determining necessary dimensions, measuring packet of sunflower seeds Math focus: Volume, Pythagorean theorem, trigonometry, composite solids
5.16	K	151.b.i, ii, iii 151.d.i 151.f.i 151.g.i, ii	LP1, LP2, LP6, LP7	Context: Elephants’ enclosure at a zoo, lithium ion battery, manufacturing packaging, spherical elephants and cylindrical snakes, comparison of human, ant, and elephant Math focus: Maximizing area, surface area and volume, raw and relative strength, surface area to volume ratio
5.17	K	151.c.ii 151.f.i 151.g.i, ii	LP1, LP2, LP3, LP4, LP6	Context: Volume problem-solving concepts to situations involving density: pillow, anchor, pencils, number of fish in a tank, determining if a lime will float or sink

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
				Math focus: Volume and density, unit conversions
5.18	K This lesson is very similar to Lesson 5.5.	151.b.i, ii, iii 151.g.i, ii	LP1, LP2, LP6, LP7	Context: Making balloons for parades with restrictions, determining the amount of helium needed Math focus: Unit conversions, scale factor, volume, graphing, equations
Modeling Task- <u>A</u> <u>New</u> <u>Container</u>	M Consider opening the prompt by asking students to define the problem they want to solve as well as giving students the opportunity to identify quantities of interest to get to the full aspects of CPE 151.a. (Doing so may open space for other CPEs, not listed, to be included depending on the figures students choose.)	151.a.i 151.b.i, ii, iii 151.d.i 151.f.i 151.g.i, ii	LP1, LP4, LP5, LP6, LP7	Context: Designing a container or image showing what it will look like and what its dimensions will be, including calculations Math focus: Designing a three-dimensional figure that can hold 16 fluid ounces

Unit.Lesson	Priority D - Delete M - Modify K - Keep	Content and Practice Expectations (CPE)	Learning Principles (LP)	Additional Lesson Information
Modeling Task 2- <u>Representation in the United States Congress</u>	M Add a prompt like Example 4 from Annotated Examples in M151: Modeling with Geometry.	151.a.i 151.c.i 151.g.i, ii	LP1, LP2, LP3, LP4, LP6, LP7	Context: Congress and House of Representatives Math focus: Area density based in modeling situations