

Baseline Professional Learning Handout

M204 Exponential and Logarithmic Functions and Equations

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High School - Algebra

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
 - a. Factor a quadratic expression to reveal the zeros of the function it defines.
 - b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
 - c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.151/12)^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.★

High School - Functions

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.★
 - a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
 - b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
 - c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

4. Find inverse functions.
 - a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x + 1)/(x - 1)$ for $x \neq 1$.
 - b. (+) Verify by composition that one function is the inverse of another.
 - c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
 - d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
 - a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
 - b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
 - c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

M204 Content and Practice Expectations

204.a	Understand the relationship between exponential and logarithmic functions.
204.b	Graph exponential and logarithmic functions.
204.c	Interpret logarithmic and exponential functions that arise in applications in terms of the context.
204.d	Use logarithms to analyze exponential models.

M204 Content and Practice Expectations and Indicators

Content and Practice Expectations	Indicators Choose an artifact where you...
204.a: Understand the relationship between exponential and logarithmic functions.	i. use reasoning about exponents to explain the value of a logarithmic expression.
	ii. write an exponential expression to verify the value of a logarithmic expression.
204.b: Graph exponential and logarithmic functions.	i. graph an exponential function over a domain that includes non-integers.
	ii. graph a logarithmic function.
	iii. explain the effect of changing the parameters in an exponential or logarithmic function on its graph.
204.c: Interpret logarithmic and exponential functions that arise in applications in terms of the context.	i. interpret key features of the graph of an exponential function in terms of the context being modeled.
	ii. interpret key features of the graph of a logarithmic function in terms of the context being modeled.
	iii. explain the choice of domain for a logarithmic or exponential function in terms of the context being modeled.
	iv. calculate and explain the meaning of the average rate of change for an exponential or logarithmic function in terms of the context being modeled.
204.d: Use logarithms to analyze exponential models.	i. create an exponential model for a context.
	ii. use a logarithm to solve for an unknown value of an exponential model.

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Content and Practice Expectations	Indicators Choose an artifact where you...
	iii. interpret values for exponential models in context.

Let's Investigate

Exponentials and Logarithms I Task

Recall that $\log_b(x)$ is by definition the exponent which b must be raised to in order to yield x ($b > 0$).

Part I

- Use this definition to compute $\log_2(2^5)$.
- Use this definition to compute $\log_{10}(0.001)$.
- Use this definition to compute $\ln(e^3)$.
- Explain why $\log_b(b^y) = y$ where $b > 0$.

The above technique can be used to raise numbers to logarithmic powers by first simplifying the exponent.

Part II

- Evaluate $10^{\log_{10}(100)}$.
- Evaluate $2^{\log_2(\sqrt{2})}$.
- Evaluate $e^{\ln(89)}$.
- Explain why $b^{\log_b(x)} = x$ where $b > 0$.

Let's do some math! Round 1

Task 1: Investigating Exponentials

Students are given a prompt like this:

Consider the following scenarios.

1. There are currently 289 million registered cars in the United States, increasing by about 1.5% annually.
2. For the original strand of COVID-19, one infected person was responsible for further infecting about two people during their infectious period. The spread of the virus continued in that pattern. The current US population is about 330 million people.
3. There are 20,000 fish in an infected pond. The population of fish declines to 8,000 in four hours.

For each of the scenarios above, write down questions you could ask about the situation. Write as many questions as you can think of.

As a group, you will choose some interesting questions to answer. If you would need to do calculations, make a sketch, or draw a graph in order to answer a question, then it's probably an interesting question.

Write a report to explain your questions and answers. The report should include:

- a context for your situation. Write a story that includes all the relevant information. If the description of the situation is missing some details, fill them in however you want.
- an equation to describe the situation, with an explanation of what each part of the equation represents.
- a labeled graph of the situation, including the questions you chose and your answers. Include your process for finding the answers. Say what the answers mean in the context of this situation.

Adapted from [Illustrative Mathematics Modeling Prompt](#) (Illustrative Mathematics, 2019)

Sample Learning Experience

Provide time for students to consider the question posed and independently develop a response before they are encouraged to work with a partner or small group.

Monitor and select students to share their approach as they work on these prompts. Consider using the following questions to facilitate the discussion:

- How would you describe the relationships captured in your preliminary work: linear, exponential, or something else? How can you tell?
- What's promising about this approach?
- What questions does this approach surface for you?
- What can you take from this approach to apply to your own work?

Provide time for students to revise and refine their approach. Give students additional time to explore their ideas and conduct additional research, as well as to prepare their report. Reports can take any form that students feel is

appropriate: traditional presentation, TED Talk style, a video, an animation, or any other form that makes sense to them.

As students showcase their work, invite classmates to engage with the presentation. Consider asking the following questions:

- *What is something you appreciate about the presentation?*
- *What about this reasoning makes sense?*
- *Is there anything you disagree with?*
- *What questions surface for you?*

Provide time for students to revise their work.

Learning Principles

Engage with cognitively demanding tasks in heterogeneous settings (LP 1). Students should be given opportunities to grapple with multistep, non-routine tasks that promote mathematical rigor. These experiences should be differentiated so that all students engage in appropriate challenges—for example, through tasks with multiple entry points and solution pathways. These experiences should continue to integrate knowledge and skills developed in grades 6-8 at the level of sophistication of high school mathematics.

Engage in social activities (LP 2). Students should have opportunities to work independently and to communicate with one another about mathematics by engaging in collective and collaborative learning activities. Explaining and having opportunities to revise one’s thinking has excellent value in solidifying one’s knowledge.

Build conceptual understanding through reasoning (LP 3). Students should be given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Through reasoning and work with multiple representations, students learn why procedures work and build a conceptual understanding of key mathematical ideas.

Have agency in their learning (LP 4). Students should be able to choose tasks and learning experiences that align with their interests and aspirations. All students have rich and varied experiences and home lives. Learning mathematics should bring students’ identities and interests to the fore and build on the strengths that they bring to the learning space.

View mathematics as a human endeavor across centuries (LP 5). Students should understand that mathematical ideas emanated over time from civilizations around the world and have opportunities to explore these contributions to mathematics. Students should develop an appreciation of mathematics as a human endeavor: one in which they feel a sense of belonging, where they see themselves as mathematicians, and one that offers opportunities to broaden their ideas about what mathematics is, how it is used, and who it is for.

See mathematics as relevant (LP 6). Students should engage with mathematics in ways that authentically involve real-world situations. Problem-solving contexts should allow students to perceive mathematics as a tool for addressing the questions that arise in everyday life, as well as the ways it can model our world and address global economic, social, and environmental challenges. Students should also engage with mathematics in ways that connect to academic disciplines and future careers by doing the mathematics used by artists, designers, engineers, and other professionals.

Employ technology as a tool for problem-solving and understanding (LP 7). Research indicates that technology is a powerful tool for learning deeper mathematics by improving calculation efficiency and enabling more sophisticated analysis. Students should learn to use technology, with emphasis put on widely used tools and software, such as calculators and spreadsheets, to make sense of models. Technology use should not be limited to supporting “doing mathematics,” but should also be used as a tool for displaying and communicating results to appropriate audiences.

Task 1: Investigating Exponentials Student Work

Example 1:

Insert image of student work here

Example 2:

Insert image of student work here

Example 3:

Insert image of student work here

Let's do some math! Round 2

Task 2: Viral Load Prompt

Students are given a prompt like this:

When a virus invades our body, it gets our body to make so many copies of it that it disrupts normal functioning of our body at the cellular level and makes us sick. Watch this video from [Life Noggin- Just How Fast Does a Virus Spread?](#) to learn more. As the virus spreads to other cells in our body, the viral load in our body increases. The viral load measures how much viral RNA is present in some body fluid, such as blood or saliva. The unit of measure is in copies per mL using \log_{10} .

Part 1: Making sense of log base 10

Let's explore how this works. Study the table below.

x	10	100	1,000
$\log_{10} x$	1	2	3

1. What do you notice? How are the numbers changing?
2. How would you describe what $\log_{10} x$ is?
3. What would the graph of $y = \log_{10} x$ look like?
4. What do you anticipate this has to do with the concept "viral load"?

Part 2: Making connections to the medical field

A group of researchers set out to understand how viral load is associated with the risk of COVID-19 transmission. They found that the median viral load among the cases they looked at was " $5.6\log_{10}$ copies of RNA per mL of saliva." [source: [SARS-CoV-2 viral load is associated with risk of transmission to household and community contacts | BMC Infectious Diseases.](#)]

4. What about this expression makes sense to you at this point?
5. What questions do you have?

The medical field uses \log_{10} both as a label and in its mathematical sense. Examine the table below to see how.

$W\log_{10}$	$1\log_{10}$	$2\log_{10}$	$3\log_{10}$	$4\log_{10}$	$5.6\log_{10}$
number of copies per mL	10	100	1000		

6. Complete the table using patterns that you see.
7. The same researchers found that a small portion of cases had viral loads greater than $8\log_{10}$. How many copies per mL is that? How much greater is this viral load than $4\log_{10}$?

Part 3: Reflecting on your learning

8. What is something you've learned about the meaning of logarithms?
9. What do you think the significance of these numbers is? Why would anyone be interested in understanding the concept of viral load?

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10. *What is something you've learned about yourself, as you made sense of these ideas?*

Sample Learning Experience

Launch this experience by asking students what's the largest number they can imagine. You can use numbers they share to lead into the idea of how viruses make us sick, possibly asking: *do you think your body can hold that many copies of a virus?* Take a poll and make note of some of the ideas that surface in a visible space, like a poster or whiteboard. Reference these ideas as applicable throughout this journey. Listen for ideas that the Life Noggin video addresses as well as ideas that can transition to the concept of viral load.

Show the video and amplify the segment where the virus gets our cells to make multiple copies. Share with students that it is this phenomenon (replication of RNA) that experts in the medical field work to quantify with a unit of measure called *viral load*. Encourage students to share questions they have and, as time permits, allow students to research this topic.

Consider using these questions to debrief this segment:

- *What is something that you learned that was new?*
- *What surprised you?*

Part 1: Making sense of log base 10

Frame this learning experience by sharing that we will be working to understand how viral load is measured, and this entails very large numbers. Give students time to work on Part 1 individually, prior to inviting them to work with a partner or small groups.

As students work, consider posing these questions:

- *What do the values in the log function represent?*
- *How is the function growing? Where do we see that in the table? Graph?*
- *What connections are there between the different representations?*

Monitor and select varied approaches and responses to share out to the class. Consider using the following questions to facilitate discussion:

- *What about this makes sense to you?*
- *What questions does this work surface for you?*
- *What can you take from this approach to apply to your own work?*

Provide time for students to revise and refine their work.

Part 2: Making connections to the medical field

Give students quiet work time before inviting students to partner or work with a small group. Consider pausing after Questions 4 and 5 to give space for students to share their current understandings and work to develop a shared understanding of the context. Capture ideas students share on a visible surface such as chart paper or white board.

Encourage students to engage in the next set of questions as a way to address some of the questions they've surfaced as well as to confirm some of the ideas they have.

As students work, pose the following questions:

- *What similarities do you see between the work you did in Part 1 and what you're looking at here?*
- *How can you use what you know from Part 1 to explain what you see happening in this table?*
- *What tools might be helpful to you: equation? table? graph?*

- *What connections are there between the different representations?*

Monitor and select students to share their thinking with the class. Use suggested questions as you see fit. Look for opportunities to connect to students' initial ideas and questions captured in early parts of this experience.

Part 3: Reflecting on your learning

Give students an opportunity to reflect on their learning and to share with others. As needed, provide time for students to revise and refine their work.

Task 2: Viral Load Student Work

Example 1:

Insert image of student work here

Example 2:

Insert image of student work here

Example 3:

Insert image of student work here

Criteria for Success

Conference and Provide Revision Support	Accept with Revision	Accept
The student's artifact shows an emerging understanding of the expectations of the indicator(s). After conferencing and additional instruction/learning, the student may provide a revised or different artifact as evidence of the indicator(s).	The student's artifact is approaching a full understanding of the expectations of the indicator(s). The artifact may contain execution errors that should be corrected in revision. The student may revise the selected artifact or submit a different artifact.	The student's artifact demonstrates evidence that they have met the expectations of the indicator(s).