

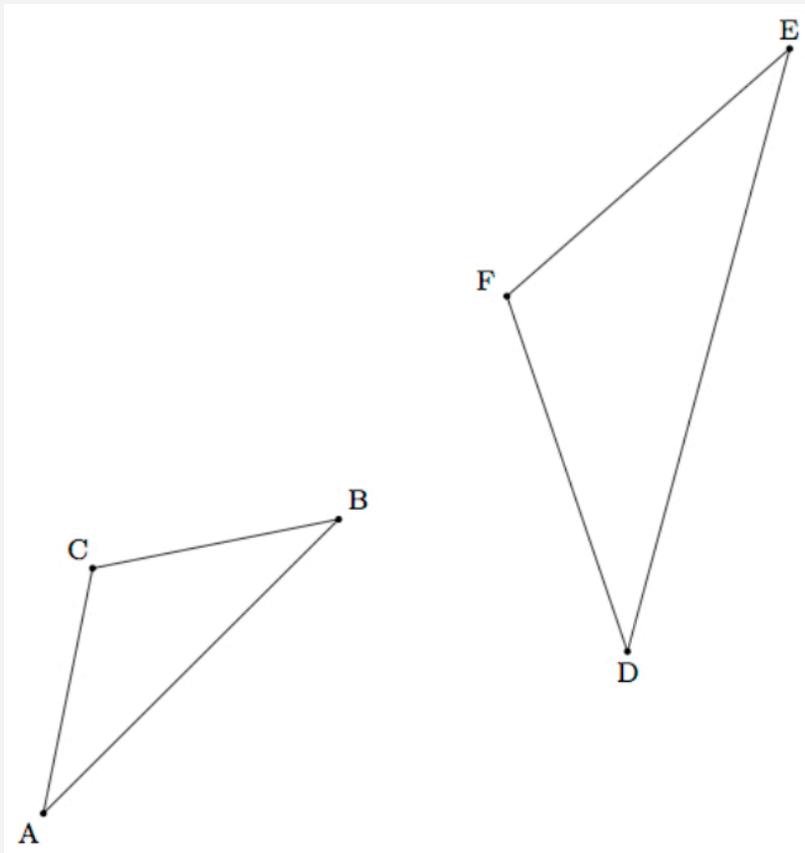
# Annotated Examples: M153 Reasoning and Proof through Similarity

The examples that follow are intended to illustrate how the learning principles are used to support students' engagement with the content and practices outlined in this badge. These examples do not provide comprehensive coverage of those expectations, but rather elevate some of the learning principles that are less likely to be part of published curricular materials for mathematics instruction. The examples that follow were developed by the Math Badging writing team, unless otherwise specified. These are a small sample of types of learning activities that can be done with students, both in and out of a traditional classroom setting.

## Example 1: Describe Your Transformations

Students are given a problem like this:

*In the two triangles pictured below,  $m(\angle A) \cong m(\angle D)$  and  $m(\angle B) \cong m(\angle E)$ ;*



*Using a sequence of translations, rotations, reflections, and dilations, show that  $\triangle ABC$  is similar to  $\triangle DEF$ .*

(Illustrative Math task Similar Triangles, 2016)

### Sample Learning Activity

Begin by giving students quiet time to make sense of the information given and process the task they have to complete. Provide them with physical tools, such as transparencies or patty paper, or digital geometry software to experiment with different transformations.

After some individual time, encourage students to discuss progress with a partner or small group. Encourage students to make their thinking visible. Initially, focus a conversation on how students interpreted and used the given information. Consider pulling the class together to discuss and share students' labels and use of tools to build shared understanding of the known information about the triangles. Monitor and select student work that aids in the conversation. Note some may mark up their paper whereas others may use their transparency or tracing paper to arrive at the conclusion that the given corresponding angles are congruent.

Monitor for, select, and share student work that embodies different approaches and levels of formality for discussion. Listen for the various ways students name the sequence of transformations. Note and select various levels of formality students are comfortable with as they share their work with each other. For example:

#### Sample Student Response 1

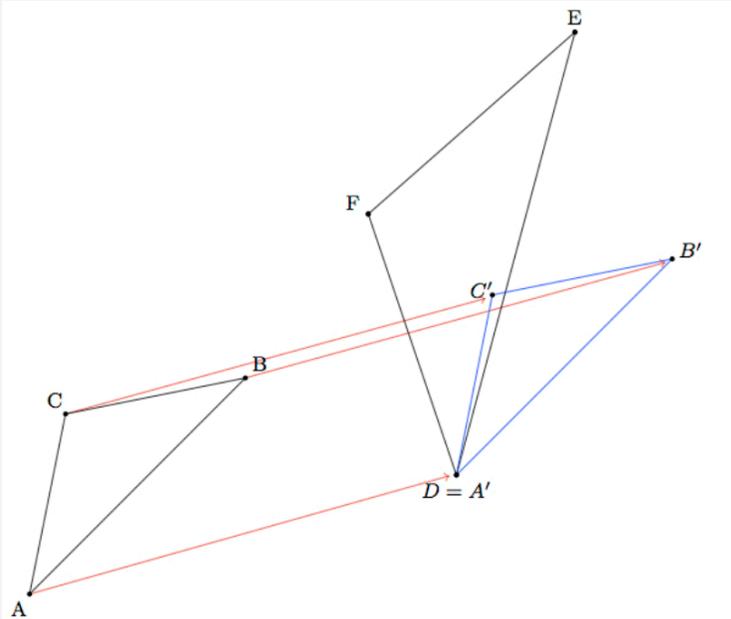
I would do a translation first, then a rotation.  
Then the dilation would be last.

#### Sample Student Response 2

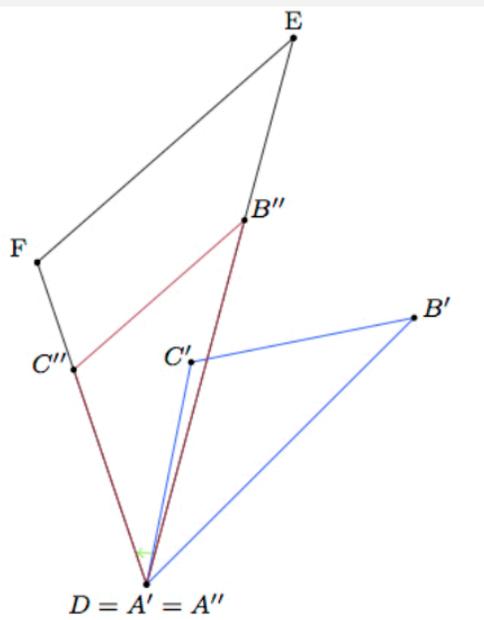
First, I would slide the small triangle over so that B is on top of E. Then you turn it a little so that the angles match up. If the angles are matched up, BA is on top of ED, and BC is on top of EF. Then you just have to stretch A so it's on top of D. Then  $\triangle ABC$  will be on top of DEF and they are similar.

#### Sample Student Response 3

Translate A to D as shown below:



Then rotate  $\triangle A'B'C'$  by angle  $C'A'F'$  as shown below:



Note that  $\overrightarrow{A''C''} = \overrightarrow{AC}$  because of the angle of rotation. Note too that  $\overrightarrow{A''B''} = \overrightarrow{DE}$ ; this is true because  $m\angle B''A''C'' = m\angle BAC$ , since rigid motions preserve angle measures, and  $m\angle BAC = m\angle EDF$  by hypothesis.

We have already moved  $A''$  to  $D$  and so we choose  $D$  as the center of our dilation. We would like to move  $B''$  to  $E$  and the dilation factor which will accomplish this is  $\frac{|DE|}{|A''B''|}$ . To check that  $C''$  maps to  $F$  note that  $m(\angle DEF) = m(\angle A''B''C'')$ . Angles are preserved by dilations and so this means that  $\overline{B''C''}$  must map to  $\overline{EF}$ .

As students share their sketches and reasoning with classmates, invite them to make sense of each other's sketches. Consider the following questions:

- What are some things you appreciate about this explanation?
- What questions does this explanation surface for you?
- What can you take from this explanation to apply to your own work?

Following the sharing and discussion, give students an opportunity to revise their work and share with each other again. Focus this second discussion on changes or adjustments that they have made based on their peers' work.

In this example, students are:

- given a cognitively demanding task. It requires students to reason using transformations that two triangles are similar. The learning activity is structured to give all students access by inviting them to write explanations with varying degrees of formality and giving an opportunity for revision after engaging with some of their peers' explanations (LP 1).
- working collaboratively through structured opportunities to ask questions to each other to better understand the thinking process (LP 2).
- building conceptual understanding through reasoning, as they must construct a logical argument based on their understanding of transformations and the given triangles (LP 3).
- encouraged to have agency, as they choose their solution pathways and are given space to reflect on how their approach might change based on engagement with their peers' work (LP 4).
- given an opportunity to use geometry software to experiment with transformations (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:

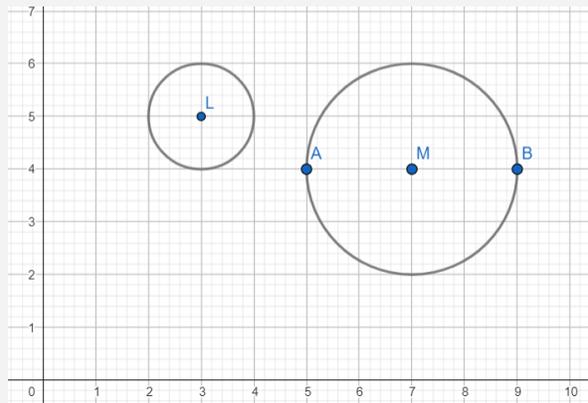


- **Critical Dialogue:** [Explaining My Reasoning](#) (FL.ID.3)

## Example 2: Dilating Circles

Students are presented with a problem like this:

The picture below depicts a dilation between Circle L and Circle M. A dilation is defined as, “A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor” ([Mathematics Glossary CCSS](#)).



1. Given the position of the images, is Circle M a dilation from inside, on, or outside Circle L?

2. Determine the scale factor of the dilation to produce Circle M.

3. Use [GeoGebra](#) or [Desmos Geometry](#) to recreate the image online. Then, find the center of dilation from Circle L to Circle M.

\*Or use this link to access the [pre-created](#)

[GeoGebra drawing](#).

4. Given a scale factor of 4 with Circle M as the preimage, what is the set of coordinates of the image of points A and B? What was the center of the dilation that resulted in those coordinates? Would a different center of dilation result in the same coordinates you found? Why or why not?
5. Given a scale factor of  $1/4$ , explain how Circle M would change. Under what conditions would this happen?
6. Revisit the definition of a dilation: “A transformation that moves each point along the ray through the point emanating from a fixed center and multiplies distances from the center by a common scale factor.” How did you use parts of the definition to respond to the questions above?

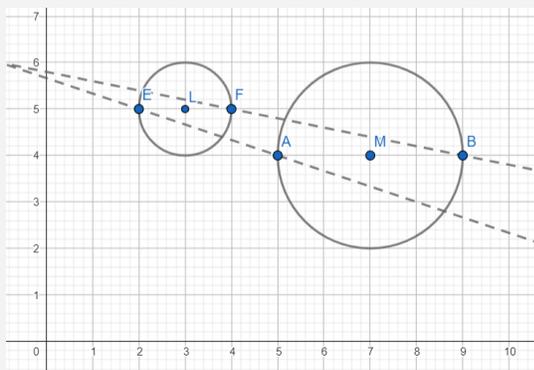
## Sample Learning Activity

Introduce the lesson by displaying the given image. Ask students: “What do you notice? What do you wonder?” Chart student responses close to the image, in a place that is visible to the whole class. Encourage them to elaborate in order to make ways they are attending to structure visible to themselves and their classmates.

Introduce the definition provided and ask students to make connections between the definition and what they shared on what they noticed. Ask: “What connections do you see between what you noticed and this definition?” Give students a minute of quiet time to think, then share with a partner before eliciting responses from the whole group. Chart student connections close to the previous collection of student-generated ideas.

If the concept of dilations is new to students, consider using excerpts from *Illustrative Mathematics Grade 8 Unit 2 Lesson 2* as a way to introduce it before engaging in this task.

Give students time to engage in Question 1 individually, followed by partner or small group work. Monitor and select varied approaches to this prompt to share for whole class discussion.

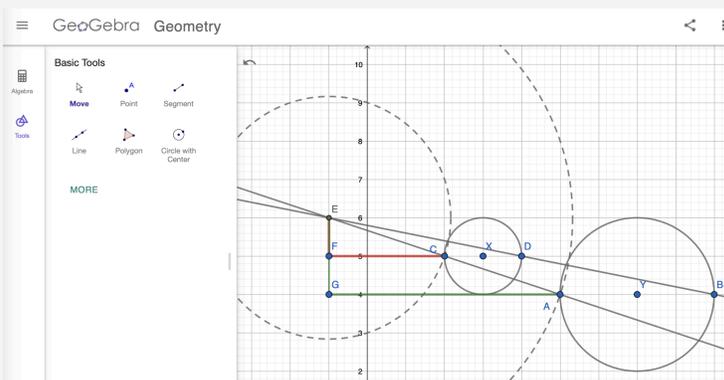


As students share their approach with classmates, invite them to make sense of each other’s work. Consider the following questions:

- *What are some things you appreciate about this approach?*
- *What questions does this approach surface for you?*
- *What can you take from this explanation to apply to your own work?*

Following the sharing and discussion, give students an opportunity to revise their work or build upon their work to share with each other again. Focus this second discussion on changes or adjustments they have made based on their peers’ work.

As students return and work on Questions 2 and 3, monitor and select varied approaches to these prompts for whole class discussion.



As they share their approach with classmates, invite students to make sense of each other’s work. Consider the following questions:

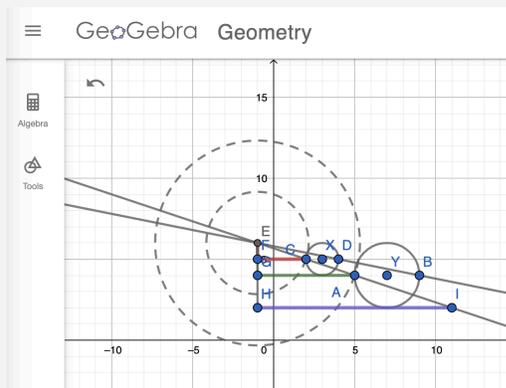
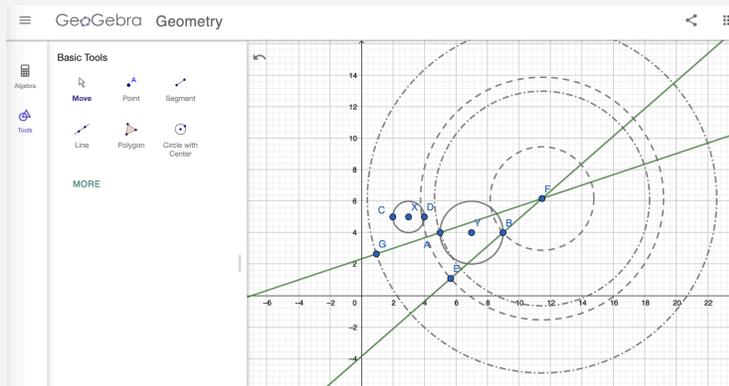
- *What are some things you appreciate about this approach?*
- *What questions does this approach surface for you?*

- *What can you take from this explanation to apply to your own work?*

Following the sharing and discussion, give students an opportunity to revise their work or build upon their work to share with each other again.

As students return and work on Questions 4 and 5, monitor and select varied approaches to these prompts for whole class discussion. Focus this discussion on the role the center of dilation plays in determining where dilated images live on the plane.

Following the sharing and discussion, give students an opportunity to revise their work or build upon their work to share with each other again.



For Question 6, consider using the [Math Language Routine 1: Stronger and Clearer Each Time](#) (MLR 1) to give students a structure with which to synthesize their learning. Begin by posing the prompt and giving students time to write their preliminary response individually. Then give students three to four minutes to share their response with a peer. Then give students a moment to write a note to themselves based on what they heard from their classmate. Give students another opportunity to pair up with a different classmate and share responses again. Finally, give students time to revise their preliminary response.

In this example, students are:

- given a cognitively demanding task. They are asked to consider how the scale factor of a dilation creates a larger or smaller image. Students also have opportunities to grapple with and understand the role the center of dilation plays in this transformation (LP 1).

- working collaboratively in pairs and sharing their findings with the class. Students are encouraged to ask questions of each other to better understand the thinking process, as well as given space to learn from their peers (LP 2).
- negotiating the meaning of dilations as they move back and forth between the definition and the actions they take in manipulating a given image (LP 3).
- given space to develop agency, as they choose their solution pathways and reflect on how their approach might change based on engagement with their peers' work (LP 4).
- using technology as a tool, as they employ straight edges and/or web-based apps to identify the elements that define a given dilation, as well as explore alternate possibilities (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Logical Thinking:** [Recognizing Patterns](#) (OT.Crit.2.e)

## References

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