

Annotated Examples: M102 Modeling with Linear Functions and Equations

The examples that follow are intended to illustrate how learning activities are used to support students' engagement with the content and practices outlined in this badge. They do not provide comprehensive coverage of those expectations, but rather elevate some of the learning activities less likely to be part of published curricular materials for mathematics instruction. The examples that follow were developed by the Math Badging writing team, unless otherwise specified. These are a small sample of types of learning activities that can be done with students, both in and out of a traditional classroom setting.

Example 1: Taxi Ride

Students are given a problem like this to interpret the parameters of a linear function:

Part 1

Lauren records her taxi fare displayed on the computer screen at different points of time during a taxi ride in the following table.

Distance, d , in miles	Fare, F , in dollars
3	8.25
5	12.75
11	26.25

How would you describe the relationship between the two quantities?

Part 2

Identify two quantities that have a relationship you'd like to track and that you believe is linear in nature. Then answer the following questions.

- *What quantities are you tracking?*
- *What about the relationship between these two quantities leads you to believe it will be linear?*
- *Create a model that describes the relationship between the two quantities.*
- *What surprised you about the relationship you just modeled?*

(Adapted from [Illustrative Mathematics, Taxi](#), 2016)

Sample Learning Activity

Set the stage by asking students, “*Why might Lauren keep track of the distances she travels and the amount she pays?*”

Ask students if they know of other situations where they might want to keep track of what they are doing and why.

Monitor students as they work on their description of the relationship between the distance and the fare Lauren paid. Select student work that illustrates different techniques: use of a graph, analysis of rates of change within the table, or the writing of an equation. Consider sequencing the work in a way that builds a coherent mathematical story and allows for connections across representations. Encourage students to ask questions to their peers as mathematical ideas are presented.

Amplify aspects of the conversation that:

- explain the linear nature of this relationship.
- contextualize the slope and y-intercept of the line.
- lead to the equation of this linear function.

Allow students the opportunity to return to their work to revise, as necessary.

During the revision stage, have students work in pairs to complete the task. Allow them the option of using chart paper, Google Slides, or other creative outlets to showcase their work.

In this example, students are:

- given a conceptual task reflecting a real-world scenario that requires students to convey their understanding. They are also asked to consider improvements to their work (LP 1).
- working collaboratively in pairs to complete the task, allowing for social interaction among peers (LP 2).
- applying their understanding of linear functions to identify and model a relationship between two quantities they believe to be linear in nature (LP 3).
- bringing their identities and interests to the fore as they identify a relationship between quantities they wish to explore (LP 4).
- using technology as a tool (LP 7).

Through the implementation of this learning activity, look for opportunities to support students’ growth and development on this XQ competency:



- **Mathematical Modeling:** [Explaining models](#) (FL.MST.2.c)

Example 2: Catch Flights

Students are given a problem like this:

While on an airplane, Leticia noticed that she could toggle back and forth between imperial and metric measurements showing data at a given time point during the flight. She took photographs of the imperial and metric data on two different flights.

Imperial and Metric Data from Flight 1

Destination METRIC Atlanta (ATL)		IMPERIAL	
Time to Destination: 1:01		Distance to Destination: 406 mi	
Local Time at Origin: 7:41 AM		Local Time at Destination: 7:41 AM	
Altitude: 28 ft		Head Wind: 0 mph	
Outside Temperature: 66°F		Ground Speed: 11 mph	

Destination METRIC Atlanta (ATL)		IMPERIAL	
Time to Destination: 1:01		Distance to Destination: 653 km	
Local Time at Origin: 07:41		Local Time at Destination: 07:41	
Altitude: 9 m		Tail Wind: 0 km/h	
Outside Temperature: 19°C		Ground Speed: 17 km/h	

Imperial and Metric Data from Flight 2

Destination METRIC Atlanta (ATL)		IMPERIAL	
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Time to Destination: 3:23	Distance to Destination: 1591 mi
Local Time at Origin: 3:40 PM	Local Time at Destination: 5:40 PM
Altitude: 4279 ft	Head Wind: 19 mph
Outside Temperature: 77°F	Ground Speed: 0 mph

Destination METRIC Atlanta (ATL)	IMPERIAL
Time to Destination: 3:23	Distance to Destination: 2561 km
Local Time at Origin: 15:40	Local Time at Destination: 17:40
Altitude: 1304 m	Head Wind: 30 km/h
Outside Temperature: 25°C	Ground Speed: 0 km/h

Sample Learning Activity

Share images with students and ask, “What do you notice? What do you wonder?”

Generate a list of what students noticed and wondered and amplify their wonderings that lead to an exploration of linear functions, such as the relationship between imperial and metric measurements of temperature, altitude, or distance to destination.

In groups, have students select a measure to focus on, like temperature, and consider how they can model the relationship between imperial and metric methods of measurement.

Allow groups time to consider the question and identify possible steps to approach the question. Encourage students to use graphing technology as they complete the task.

Bring the groups together to allow them to share ideas on how they are approaching the task. Use the following questions to discuss highlighted ideas:

- *What’s promising about this approach?*
- *What questions does this approach surface for you?*

- *Do you think the model is reasonable? Why or why not?*
- *What can you take from this approach to apply to your own work?*

Follow this intermediate discussion with the following prompt for groups to complete. Groups should reflect on and revise their thinking based on the ideas shared during the discussion.

1. *Write an equation that takes a measure from metric to imperial.*
2. *If you were to make the graph of the equation you wrote, how well does it fit the data you have? What might account for any differences you see?*
3. *For someone who starts with a measure in imperial, how might they use your equation to get the metric equivalent?*
4. *What additional questions could be answered about this context using some of the models we've seen? Are some models more helpful than others in answering these additional questions?*

Circulate among the groups and take note of various solution paths they are using to approach the problem. Identify groups that will present to the whole class. Be mindful to emphasize different approaches to the problem. Consider a sequence of presentations to build a mathematical story. Encourage students to ask questions to their peers as solutions are presented.

Possible points to emphasize during a discussion of responses to these questions:

- For question 1, look for students who organize information in a table or graph to then determine an equation. Focus discussion on eliciting student reasoning.
- For question 2, listen for students who attribute discrepancies to measurement or rounding error. The airplane's instruments may only display the whole number value.
- For question 3, engage students in repeated reasoning that leads to an equation for the inverse. Name the relationship between the two equations, metric \rightarrow imperial and imperial \rightarrow metric, as inverse from the work they produce. Ask students to reflect on the usefulness of each form.
- Push student thinking by asking them to consider the following questions:
 - *Is there a point that lies on both lines simultaneously if both equations were graphed on the same coordinate plane?*
 - *What is the significance of this point?*

In this example, students are:

- given a cognitively demanding task. It requires that they analyze a situation and determine an approach to convert between two quantities without originally specifying a particular method, which means groups can create tables and graphs, and make predictions or create equations. The exercise begins with a broadly accessible task and questions—"What do you notice? What do you wonder?"—that support learners with varied prior learning activities. Students are then guided to create a linear model and interpret that model. They are also asked to consider improvements to their model (LP 1).
- working collaboratively in groups to complete the task, allowing for social interaction among peers. Students are also encouraged to ask questions to their peers to better understand the thinking process (LP 2).

- applying their understanding of linear functions to identify and model a relationship between two quantities (LP 3).
- connecting mathematics to situations they may encounter, thereby building a lens for using math to explain and make sense of relationships between quantities in their lives (LP 4).

Through the implementation of this learning activity, look for opportunities to support students’ growth and development on these XQ competencies:



- **Mathematical Modeling:** [Explaining models](#) (FL.MST.2.c)
- **Critical Thinking** [Making Connections](#) (OT.Crit.3.a.3)

Example 3: Gettin’ to the Bag

Share a table that outlines median monthly rent by state, like the one below.

Ask: “What do you notice? What do you wonder?”

State	Annual Mean Wage (All Occupations)	Median Monthly Rent	Value of a Dollar
Alabama	\$46,840	\$998	\$1.16
Alaska	\$61,760	\$1,748	\$0.95
Arizona	\$53,400	\$1,356	\$1.04
Arkansas	\$44,780	\$953	\$1.17
California	\$65,740	\$2,518	\$0.87
Colorado	\$60,840	\$1,927	\$0.98
Connecticut	\$65,450	\$1,803	\$0.94
Delaware	\$56,700	\$1,435	\$1.01
Florida	\$50,020	\$1,590	\$0.99
Georgia	\$51,940	\$1,262	\$1.08
Hawaii	\$58,190	\$2,481	\$0.85

(Source: [The Average Cost of Living by State and Why Ignoring It Could Sink Your Business](#))

Your job is to explore how different income structures impact saving for an emergency fund. Consider the following situation.

Paula works as an electrician and makes about \$52,000 a year. Her living expenses are about \$2,000 a month. She has saved \$12,000, which she estimates is enough for 6 months of expenses in case of emergency.

- A. Write an equation that represents how much Paula has after x months.
- B. Graph this equation using Desmos.com or GeoGebra.org.
- C. Describe what the graph looks like.
- D. Why does it make sense that the graph looks like that?

(Source: [25 Jobs That Pay \\$50K a Year Without a Degree](#))

Irfan works as a machine assembler for \$17 an hour and has about the same cost of living as Paula. He works full-time, which is 40 hours a week. He would like to save for 6 months of expenses.

- A. Write an equation that represents how much Irfan has after x months.
- B. Graph this equation using Desmos.com or GeoGebra.org.
- C. Describe what the graph looks like.
- D. Why does it make sense that the graph looks like that?

(Source: [Find Jobs: Assembler](#))

(Additional Resource: [Beyond the Farm: Rural Industry Workers in America](#))

Consider Paula and Irfan's income structures, living expenses, and their wish to have emergency fund savings. Use features of your graphing software to explore the following:

- Under what conditions will Irfan be able to save \$12,000?
- Explore the legal requirements for working overtime. Resource: [Overtime Pay](#)
- How many additional hours a week (overtime) will Irfan need to work if he gets paid at a rate of time and a half? Or double his regular rate?
- Prepare to share about the differences you notice between the two workers.

Next, explore income structures for jobs in a field of interest to you. One possible resource is [May 2021 National Occupational Employment and Wage Estimates: United States](#)

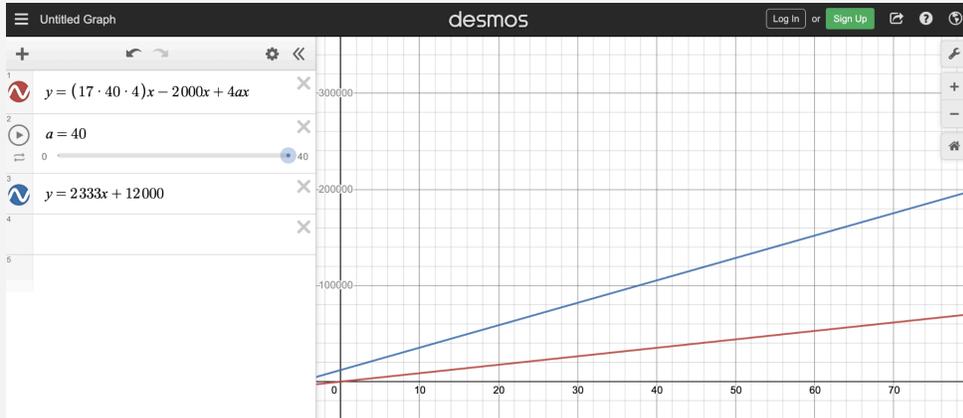
Develop a plan for saving for 6 months of expenses for an emergency.

- What are the typical living expenses for your area?
- What is an equation that models how much money you'd have after x months?

Sample Learning Activity

Begin by giving students an opportunity to notice and wonder about data found on a median monthly rent table. Build on their wonderings to introduce the idea of monthly living expenses. Ask students to identify a state where \$2,000 would be a reasonable estimate of monthly living expenses.

Give students time to begin exploration independently, then invite them to work in pairs to engage in the exploration. As students work, monitor for ways they communicate their reasoning. Some students may use features of the graphing software to focus on particular points of interest, while others may create a table of values to compare and illustrate the changes they see.



Facilitate the discussion about the exploration. Here are some points to emphasize:

- Connections between the context, features of the graph (slope and y-intercept), and impact on the graph as constraints and assumptions change. For example, if Paula gets a raise, the line has a greater slope and the line is steeper. The rate of change is also greater.
- Encourage students to compare the structure of the resulting equations, when parameters changed, with the structure of the original equation they determined in A.

Give students additional time to revise their work.

In this example, students are:

- given a cognitively demanding task. The exercise begins with a broadly accessible task and questions—“What do you notice? What do you wonder?”—that support learners with varied prior learning activities. Students are asked to explore and explain how changes in parameters impact the graph of a linear equation, as well as how those changes connect to a real-life context (LP 1).
- working individually and collaboratively as they make progress through this exploration. Students are encouraged to ask questions to their peers to understand one another’s thinking process (LP 2).
- given a task that centers reasoning with multiple opportunities to make sense of the impact of manipulating parameters in a linear equation on its graph. Students build in-depth understanding as they work to justify these changes (LP 3).
- given access to a graphing tool to aid in their understanding of and reasoning with multiple representations of the equation of a line (LP 7).

Through the implementation of this learning activity, look for opportunities to support students’ growth and development on these XQ competencies:



- **Mathematical Modeling:** [Explaining models](#) (FL.MST.2.c)
- **Critical Thinking** [Making Connections](#) (OT.Crit.3.a.)

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