

Annotated Examples: M154 Coordinate Geometry

The examples that follow are intended to illustrate how the learning principles are used to support students' engagement with the content and practices outlined in this badge. These examples do not provide comprehensive coverage of those expectations, but rather elevate some of the learning principles that are less likely to be part of published curricular materials for mathematics instruction. The examples that follow were developed by the Math Badging writing team, unless otherwise specified. These are a small sample of types of learning activities that can be done with students, both in and out of a traditional classroom setting.

Example 1: Micro-Mobility

Students are given a problem like this:

Cities have begun to offer micro-mobility options like shared electric scooters and bikes in hopes of meeting the transportation needs of their communities. For this task, you will explore the limits and possibilities of using such devices.

Use Google Maps to locate at least three places of interest. Some options could include your school site, a place you go to often, or a place of work. Use technological tools, like [Desmos](#) or [GeoGebra](#), or transparency paper to superimpose a coordinate system over your map.

- 1. If you were to use an electric scooter that can travel 35 miles on a single charge, would you be able to make the trip from the first place of interest to the second, and return back to the first point of interest? Justify your response using words, numbers, and symbols.*
- 2. How long would it normally take to make that round trip using other modes of transportation you have access to (car, bus, or walking)? How long do you estimate it would take to make the trip on a scooter?*
- 3. What are the advantages and disadvantages of offering micro-mobility options?*

Sample Learning Activity

Give students an opportunity to begin the task independently, then invite them to work in pairs to share ideas and discuss strategies for completing the task. As students are working, monitor and identify solution paths to share with the whole class. Note some students may superimpose a coordinate system using tracing paper or web-based applications like [Desmos](#) or [GeoGebra](#). Some students may center their map, as illustrated below, while others may position it in the first quadrant.

Here are some questions to facilitate the discussion of Question 1:

- *What do you appreciate about this approach?*
- *What questions does this approach surface for you?*
- *What are some commonalities and differences among the various approaches shared?*
- *How might you determine if there's enough charge to make the round trip? What information do you need to figure out in order to be confident about your answer?*
- *What can you take from this approach to apply to your own work?*

Provide additional time for pairs to revise their work and/or continue with completing the task using the ideas that surfaced.

As students start Question 3, consider offering tools for them to conduct research. Here are a few websites where they can learn more about micro-mobility:

- [Micro mobility: A Travel Mode Innovation | FHWA](#)
- [The Environmental Impacts of E-Scooters](#)
- [Micro mobility: moving cities into a sustainable future | EY](#)

Offer students the opportunity to work in pairs or small groups to complete the task. Give the option of using chart paper, Google Slides, or other creative outlets to showcase their work. Give students the opportunity to return to their work to revise, as necessary.

In this example, students are:

- given a cognitively demanding task. It requires students to analyze a situation, create a graphical representation of a local map, and determine the distance between identified points of interest.

This activity encourages students to reason about using graphs and determining distances to solve problems (LP 1).

- working individually and collaboratively in pairs to complete the task, allowing for social interaction among students. They are also encouraged to ask questions to better understand the thinking process (LP 2).
- building conceptual understanding through reasoning, as they decide how to situate a coordinate system onto a local map and whether a round trip is possible by determining distances. Students also analyze the benefits of using micro-mobility modes of transportation by determining how long it would take to make the round trip (LP 3).
- encouraged to have agency, as they choose the points of interest on their trip with the scooter, select approaches to their solution pathway, and make their own simplifying assumptions (LP 4).
- given space to reflect on how the use of a scooter to make a potential trip impacts themselves and others (LP 5).
- engaging with information from an online source and potentially leveraging a graphing application to complete the task (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Problem solving:** [Creating Solutions](#) (OT.PS.2a)

Example 2: The Roots of Land Surveying

Students are given a problem like this:

Part 1

According to Merriam-Webster's online dictionary, surveying is defined as "a branch of applied mathematics that is concerned with determining the area of any portion of the earth's surface, the lengths and directions of the bounding lines, and the contour of the surface and with accurately delineating the whole on paper" ([Merriam-Webster Dictionary](#)).



The image above is that of an ancient Egyptian map known as the Turin papyrus map. It dates back to about 1150 B.C. (A.F. Stone p. 117) and was discovered between 1814 and 1821 (Harrell).

- Study the map and consider what things it might capture. Read more about this map at [Turin Papyrus Map from Ancient Egypt](#). How much of what you anticipated surfaced in the reading? What surprised you?

Part 2

Read the following excerpt from *Crest of the Peacock* (Joseph, 2010):

It is worth remembering that up to 1350 BC the territory of Egypt covered not only the Nile Valley but also parts of modern Israel and Syria. Control over such a wide expanse of land required an efficient and extensive administrative system. A census had to be taken, taxes collected, and large armies maintained. Agricultural requirements included not only drainage, irrigation, and flood control but also the parceling out of scarce arable land among the people and the construction of silos for storing grain and other produce. Herodotus, the Greek historian who lived in the 5th century BC, wrote that

Sesostris [Pharaoh Ramses II, c. 1300 BC] divided the land into lots and gave a square piece of equal size, from the produce of which he exacted an annual tax. [If] any man's holding was damaged by the encroachment of the river ... The King ... would send inspectors to measure the extent of the loss, in order that he might pay in future a fair proportion of the tax at which his property had been assessed. Perhaps this was the way in which geometry was invented and passed afterwards into Greece (Herodotus, 1984, p. 169).

He also tells of the obliteration of the boundaries of these divisions by the overflowing Nile, regularly requiring the services of surveyors known as harpedonaptai (literally "rope stretchers").

Rope stretchers used ropes knotted at regular intervals as a measurement tool in their line of work.

1. For this part of the task, you will use tools of ancient rope stretchers, string, and a stick, to equally divide a rectangular piece of land into individual square lots. The land is bordered by a river along one side. Work to develop a general approach to this task.
2. Imagine the water level rises and floods part of the land. How would you go about redistributing the land to ensure each individual is again allotted an equal-sized square?

Sample Learning Activity

Part 1

Invite students to read the definition of *surveying* and ask:

- Why might someone want to determine the area, lengths, directions of bounding lines, and contours of land on paper?
- How would you describe what surveying is? What do you think the purpose is?

Then share the image of the Turin Map (source [Turin Papyrus Map from Ancient Egypt](#) by Dr. James A. Harrell). Ask students to share what they notice and what they wonder about what surveyors captured on this map. Give them time to explore additional resources to confirm their predictions and to surface additional questions. Listen for students who may ask: “How did ancient Egyptians take measurements?” and amplify that question to frame the next part of the task.

Offer students additional resources to further explore cartography and land surveying in ancient Egypt:

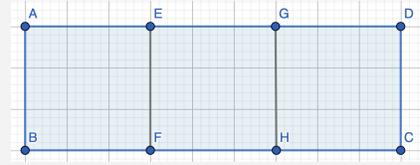
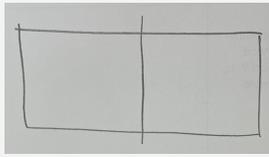
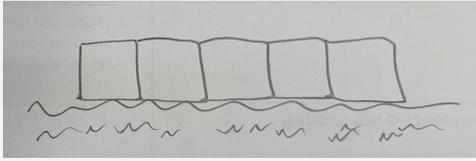
- [Surveying in Ancient Egypt](#)
- [A.F. Stone: Egyptian Cartography](#)

Part 2

Invite students to read the excerpt from *Crest of the Peacock*. Facilitate the reading by using a [3 Reads protocol](#):

- Read 1: Read the passage aloud and ask students to share what this reading is about. Record responses in a space where the class can reference.
- Read 2: Read the passage a second time and ask students what the land might look like after Sesostris’ surveyors do their job. Encourage them to brainstorm with a partner as they draw a sketch. Select a couple of sketches to share with the class. Invite classmates to ask clarifying questions and revise.
- Read 3: Read the passage a third time and ask students to anticipate how someone might go about deciding how to distribute the land. Record responses in a space where the class can reference.

Here are some sketches that may surface:



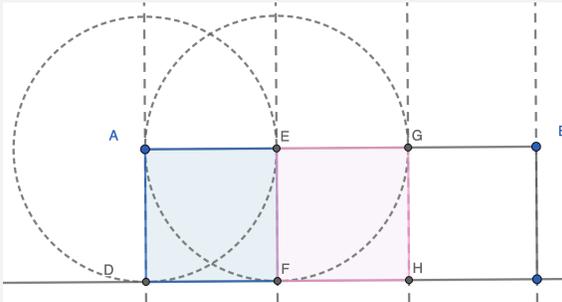
Give students time to work on Question 1. Ensure they have access to string and a straight edge of sorts (instead of a stick).

Monitor and select approaches that simulate use of a compass and straight-edge constructions. Invite students to share and learn more about each other's approaches. Ask:

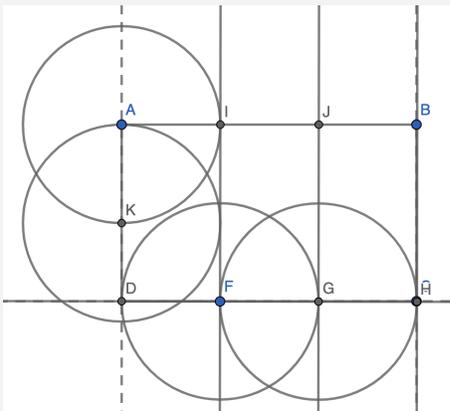
- *What's promising about this approach?*
- *What questions does this approach surface for you?*
- *Do you think the model is reasonable? Why or why not?*
- *What can you take from this approach to apply to your own work?*

Once a student surfaces an idea that brings to mind the use of a compass, encourage students to consider using tools like [GeoGebra.org](https://www.geogebra.org) to continue exploring approaches to this prompt.

Here are some possible approaches that may surface:



(Link to file: <https://www.geogebra.org/geometry/czrz7vnd>)



(Link to file: <https://www.geogebra.org/geometry/sm5v88cc>)

Discuss approaches to Question 1 before giving additional time to approach Question 2. Consider posing these questions for reflection between the two prompts:

- *What changes when the water level rises?*
- *What impact does that have?*
- *What about the approach you've developed do you think will still hold?*
- *What might you need to reconsider?*

Give students time to test their ideas and to develop responses before holding a whole class discussion. Monitor and select various approaches to these prompts.

Give time for students to revise and modify their responses.

In this example, students are:

- given a cognitively demanding task. It invites students to use the context of dividing a rectangular piece of land into equal-sized squares. In doing so, students will work to locate a point on a directed line segment between two points that partitions the segment in equal-sized segments (LP 1).
- set up to have social interactions, as they work in small groups and discuss their work with each other. These interactions may happen live in a physical classroom or virtually (LP 2).
- given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Students engage in iterative cycles of exploration as they work to solve the problem (LP 3).
- able to explore skills and dispositions as they work to solve a complex problem. As they engage in this work, students have the opportunity to develop identities and interests in using mathematics to solve problems that require making deliberate simplifying choices in order to arrive at a solution (LP 4).
- able to view mathematics as a human endeavor across centuries. In exploring artifacts and the history of ancient Egyptian surveyors, students have the opportunity to see the use of simple tools for developing mathematical solutions to complex problems (LP 5).
- using technology as a tool, as they employ graphing software, string, and/or compasses to perform their analysis (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Problem solving:** [Creating Solutions](#) (OT.PS.2a)

Example 3: The Marchetti Constant

Students are given a problem like this:

Cesare Marchetti, a physicist and systems analyst, proclaimed that people are willing to commute from their homes for about half an hour each way. This idea is known as the Marchetti Constant.

Ancient Rome, around 275 CE, was a “walking city,” meaning that the main mode of transportation was by walking. The map below illustrates an outline of the city during that period.



(Source: [The Commuting Principle That Shaped Urban History](#) - Bloomberg)

Use geometric concepts and tools to determine the degree of accuracy of the model outlined in purple in the diagram above. Aspects to explore and express in mathematical terms are as follows:

1. What is the approximate diameter of the city? Why does that make sense, in terms of what you know about typical walking rates?
2. What is the approximate area of ancient Rome?

Running water into homes was a luxury in ancient Rome. Therefore, Roman citizens were provided potable water by way of strategically placed water fountains throughout the city. It is believed that water fountains were located within a 50-meter radius from each other ([Water System of Ancient Rome](#)).

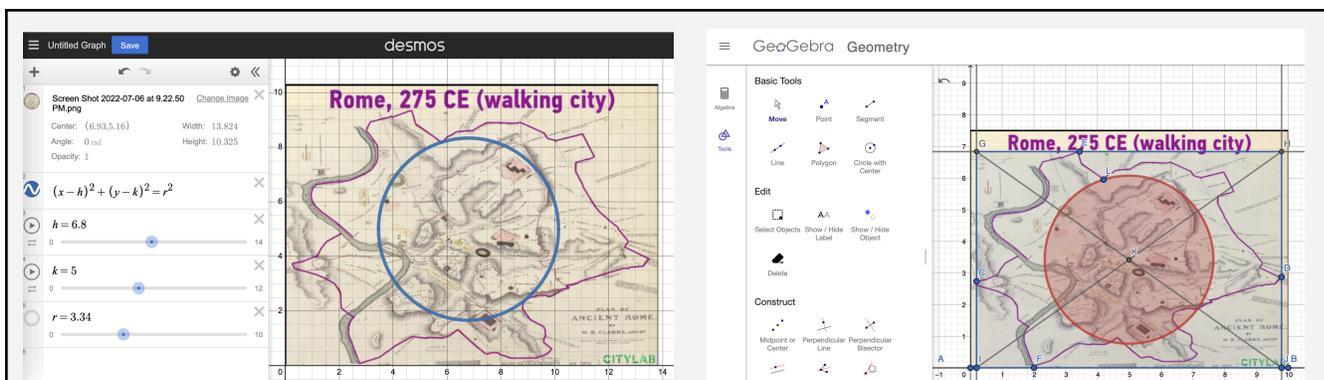
3. Based on the work you've done so far, about how many water fountains were present in the city at that time?

Extension

Explore the historical roots of the city you live in. How has the radius of the city changed over time? Does the Marchetti Constant hold?

Sample Learning Activity

Give students an opportunity to begin the task independently, then invite them to work in pairs to share ideas and discuss strategies for completing the task. As student pairs work, monitor and make note of solution paths to share with the whole class. Note that some students may superimpose a coordinate system using tracing paper or web-based applications like [Desmos](#) or [GeoGebra](#).

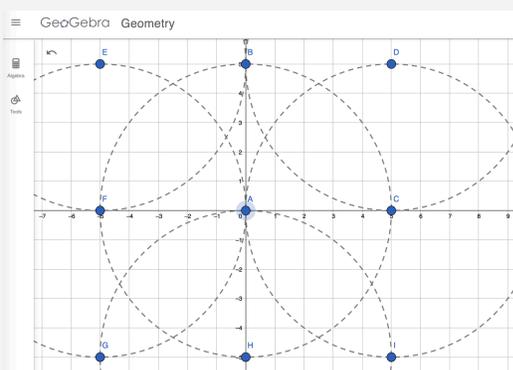


Here are some questions to facilitate the discussion of Questions 1 and 2:

- *What do you appreciate about this approach?*
- *What questions does this approach surface for you?*
- *What are some commonalities and differences among the various approaches shared?*
- *How might you determine the approximate area of the city?*
- *What information do you need in order to reach some conclusions?*
- *What can you take from this approach to apply to your own work?*

Provide additional time for pairs to revise their work and/or continue with completing the task using the ideas that surface.

As students start Question 3, monitor and select approaches they take to make sense of this. One sample approach is illustrated below. Encourage students to share and challenge approaches.



Provide additional time for pairs to revise their work and/or continue with completing the task using the ideas that surface.

In this example, students are:

- given a cognitively demanding task. It invites students to explore the idea of the Marchetti Constant and use their knowledge of coordinate geometry, circles, area, and rates to make sense of its application (LP 1).

- set up to have social interactions, as they work in small groups and discuss their work with each other. These interactions may happen live in a physical classroom or virtually (LP 2).
- given the opportunity to reason, justify, and problem solve with critical thinking, reading, writing, speaking, and listening. Students engage in iterative cycles of exploration as they work to apply their knowledge of circles and polygons, area, ratios, and coordinate geometry (LP 3).
- able to explore skills and dispositions as they work to solve a complex problem. As students engage in this work, they have the opportunity to develop identities and interests in using mathematics to make sense of real-life contexts and are required to make deliberate simplifying choices in order to arrive at a model (LP 4).
- able to view mathematics as a human endeavor across centuries. In exploring artifacts and the history of ancient Rome, students have the opportunity to see the use of simple tools for developing mathematical solutions to complex problems (LP 5).
- using technology as a tool, as they employ graphing software, string, and/or compasses to perform their analysis (LP 7).

Through the implementation of this learning activity, look for opportunities to support students' growth and development on this XQ competency:



- **Cultures:** [History and Culture](#) (FK.SS.4c)

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