

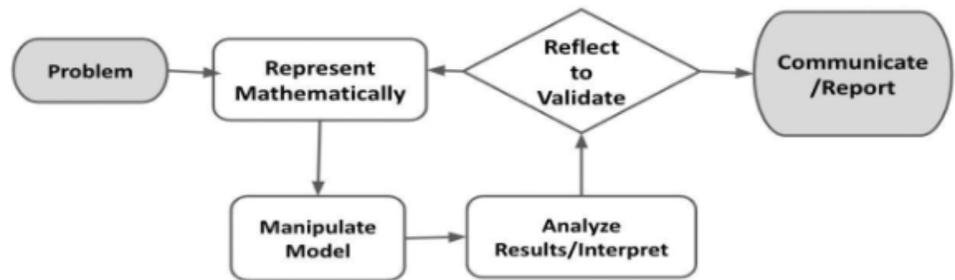
# M104B Performance Task

## Preventing Pandemics

Modeling with Functions of Exponential Type  
Teacher Version

### Design Notes

In this performance task, students will demonstrate understanding of exponential growth and exponential decay using the COVID-19 pandemic as context. They will create mathematical models by calculating the mobility of the virus, explore how different preventative measures impact the spread, and synthesize and present their learning in the form of a press briefing. You will be responsible for encouraging collaboration, facilitating student discussion and facilitating intentional check points for review and support. The learning experience as designed centers around the CCSSM modeling cycle as illustrated to the right.



Source: Diagram of Modeling Process. From "Progressions for the Common Core State Standards for Mathematics"

### Connections to Learning Principles

Students will:

- demonstrate understanding of exponential growth and decay within the context of the COVID-19 pandemic (LP6);
- explore and discuss the task collaboratively, make collective decisions about the mathematics, and present findings as a press briefing (LP2);
- model the spread of COVID-19 for 0, 5, 10, 20, 30, and  $x$  days using a table and graph (LP1);
- use the mathematical model to calculate, analyze, and interpret specific values, describing them within the context of COVID-19 (LP3);
- manipulate and interpret the features of an exponential graph that demonstrates how COVID-19 infections grow/decline as a response to preventative measures (LP7); and
- produce a clear and concise press brief using the knowledge they collected and the data explored in this task (LP4).

The following pages provide teacher versions of this task with guidance, suggestions, and comments in **red** and sample student responses in *blue italics*.

## Student Task

### The Modeling Cycle

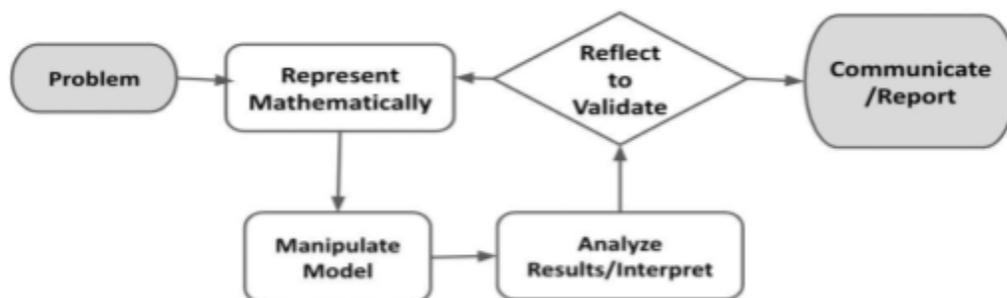
Our world has experienced many pandemics. We are currently trying to eradicate COVID-19. The first reported cases of COVID-19 happened on December 31, 2019, in Wuhan, China. Over the next few months, we would see COVID-19 grow to be a global pandemic. By March 2020, cases in the United States were on the rise. The rate of infection was fast and can be explained with mathematical models.

To follow the course of a disease, we need to know how many people, on average, are infected by an infected person if no precautions are taken. This basic **reproduction number**, or **basic reproduction rate**, is called  $R_0$ .<sup>1</sup>  $R_0$  is about 4 for COVID-19. If mutations occur that make the virus more contagious, the  $R_0$  value increases accordingly. In past infectious outbreaks, the  $R_0$  value for polio was between 10-12; for measles, it was between 12–18. In addition to  $R_0$ , the time,  $D$ , during which the infected person is infectious also plays a role. For COVID-19,  $D$  is about 5 days. For COVID-19, on average, one infected person infects four other people within five days. Watch [Video 1](#) and [Video 2](#) to understand more about what exponential growth means in terms of COVID-19.

The fast rate of infection of COVID-19 prompted the federal and state governments to institute a shutdown of all non-essential activities. Government agencies suggested other preventative measures such as vaccinations, social distancing, masking, increased sanitation and hygiene and other strategies. These preventative measures were intended to “flatten the curve” of new COVID-19 cases.

In this task, you will consider the question, “**How does exponential growth and exponential decay explain the spread and slowdown of COVID-19?**” You will use the models and your takeaways from this task to act as an immunologist and craft a persuasive message the President will use in a press conference. The message should help educate the public about stopping the spread of infection and mitigating future infectious disease and virus outbreaks.

As mentioned already, these rates can be explained with mathematical models. Mathematical models are used to make sense of ideas and phenomena in our world. As illustrated in the image below, the modeling cycle is a process of collecting data about an idea or problem, creating models of that data, analyzing the story told by those models, and reporting the results to the appropriate audience. In this task, you will engage with this mathematical modeling cycle to analyze data, evaluate findings, and identify growth and decay of a global pandemic.



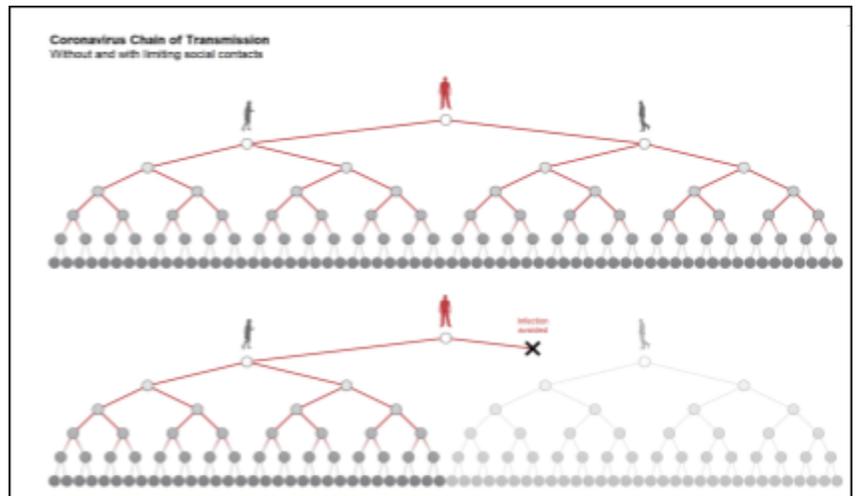
Source: Diagram of Modeling Process. From “Progressions for the Common Core State Standards for Mathematics”

<sup>1</sup> <https://www.news-medical.net/health/What-is-R0.aspx>

## Preventing Pandemics

### Launch

As the pandemic grew, the numbers associated with the virus grew. Researchers had more data to work with to help identify things like chains of transmission, or how people were spreading the virus to others, and lenses through which to examine the data, like considering whether social contact was being limited or not and how that influenced the spread of the virus. The graphs below are the visual results of some of that data. Inspect the graphs and answer the questions that follow to demonstrate your mathematical, research-scientist thinking.



Source: Growth of COVID-19 Cases & Decay of COVID-19 Cases.

### QUESTION 1



What's going on in the graphs above? Write a headline (think catchy Instagram post) that captures, in your own words, each graph's main idea.

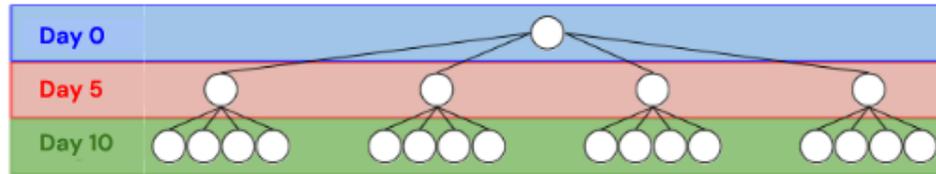
*Answers will vary. Possible student responses:*

- *The graphs show how many people can contract the virus from others, or, as the graph title says, the "Coronavirus Chain of Transmission." The first graph shows the effects, or chain, if there are no limits on social contacts, and the second graph shows how limiting social contacts can break that chain of transmission.*
- Headlines:
  - *COVID-19 Is Spreading Fast but Can Be Slowed*
  - *Slow the Spread!*
  - *One Prevention Saves Many Lives!*
  - *Avoiding at Least One Case of COVID-19 Significantly Reduces Numbers of Cases*
- **Possible Facilitation Options**
  - Present students with Chain of Transmission graphic. Give them time to discuss the image with a partner or table group before a class share out.
  - Present students with Chain of Transmission graphic. Ask, "What do you notice? What do you wonder?" Give silent think time before a class share-out and record responses on the board.
  - Present students with the Problem section without any prompting. Have them grapple with and discuss the image with their partner or table group.
- Encourage students to synthesize both figures into one headline.
- Encourage students to develop headlines that communicate the key idea in a few words that would attract readers.

## QUESTION 2



Analyze this diagram. In your own words, what is it communicating?



Source: Day-by-Day Growth of COVID-19 Cases.

Answers will vary. Possible student responses:

- Rate of spread of COVID-19 over 5 or 10 days
- The number of COVID-19 cases per five days
- How COVID-19 cases are multiplying

### Evidence of Learning

- Ensure students understand the significance of the days being represented on the graphic.
- Students should understand that only 10 days are shown, but the layers could continue indefinitely.

### Assessing/Advancing Questions

- What does each row represent?
- How much time has passed in this image?
- What doesn't this image show?

## Problem, Formulate, and Represent Mathematically

In the **Problem, Formulate, and Represent Mathematically** sections, you will examine the growth rate of COVID-19, model its spread using a table, graph, and a formula, and apply that model to calculate and make predictions about new infections and the growth rate. The **Problem, Formulate, and Represent Mathematically Checklist** will be used to ensure you have the necessary components before moving on to the remaining parts of the task.

### Problem

In dealing with a major worldwide pandemic, it is important to understand not only the number of people who are affected, but also the number of people who might become affected and by when.

Understanding these things depends on determining **how** the numbers are changing mathematically, **when** they are changing, and **why** they are changing the way that they are, among other factors. The following steps will help you begin to identify this rate of growth and be on your way to understanding and helping solve problems like this in the future!

## Formulate and Represent Mathematically

### ACTION 1



**Recall or review** the information in the launch and in the graphs presented in the launch.

Considerations

→ **How**—*mathematically*—did the cases of COVID-19 grow before the preventative measures were put in place?

### QUESTION 3



Complete the table below.

Time in Days:	0	5	10	20	30	$x$
Number of newly infected people	1	4	16	256	4096	$4^{\frac{x}{5}}$
Total number of people infected	1	5	21	277	4373	

### QUESTION 4



Create a **formula** to calculate the number of newly infected people for any number of days,  $x$ .

*Formula:  $f(x) = 4^{\frac{x}{5}}$ , where  $x$  = number of days since the first infection and  $f(x)$  = number of newly infected people.*

Students apply the  $R_0$  for COVID-19 to calculate the data for the table.

#### Evidence of Learning

- Students should understand the equation for an exponential function,  $f(x) = a \cdot b^x$ , where  $a$  is the starting value and  $b$  is the growth rate.
- Students should grapple with the question: **What is the most efficient method for calculating the number of newly infected people?**

#### Assessing/Advancing Questions

- How can we organize the number of newly infected people each day?
- What does “ **$x$  amount of days**” mean?
- Is there anything consistent about our growth?

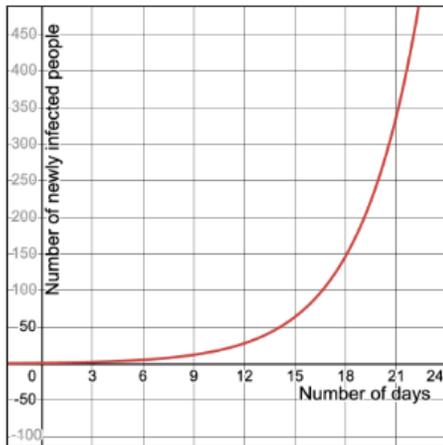
**QUESTION 5**



Create a graph to visually represent the growth of new COVID-19 cases over a 3-week period.

Use current population data to determine the number of days it would take for the number of newly infected people to equal the population of our —

- Class
- School
- State



Answers will vary depending on the population of the class, school and state. Check student responses using the given equation  $4^{x/5}$ .

Sample student response:

- Illinois Population: ~ 13 million
  - In 59 days the number of newly infected people would equal the population of Illinois.  $4^{59/5} = 12,714,752$

**Evidence of Learning**

- Students will research the school and state population, and numbers may differ.
- Students will make connections between exponents and large growth; common misconception may be understanding  $(x/5)$  represents the infection rate.
- Student methods of graphing might differ (Desmos, paper, etc.).

**Assessing/Advancing Questions**

- How does the table from the previous question and the graph from this question compare?
- Why did we not graph all of the days from the table?
- For the answer to the last question, how can you “check” your answer using the table from the previous question?

**Problem, Formulate, and Represent Mathematically Checklist**

Requirement	Teacher Notes
I modeled the spread of COVID-19 for 0, 5, 10, 20, 30, and x days using a table.	(Teachers, please add your notes here.)
I modeled the spread of COVID-19 over a 3-week period using a graph.	(Teachers, please add your notes here.)
I calculated and explained when the mathematical model would reach the population of the state.	(Teachers, please add your notes here.)
(Teachers, please add your notes here.)	

## Manipulate Model and Compute, Interpret, and Validate

In the **Manipulate Model and Compute, Interpret, and Validate** sections, you will consider how measures to limit the spread of the virus influence mathematical models of its growth and use formulas and a digital modeling applet to visualize, interpret, and then validate these changes using mathematical terminology and referencing specific variables. The **Manipulate, Compute, Interpret, and Validate Checklist** will be used to ensure you have the necessary components before moving on to the final part of the task.

### Manipulate Model and Compute

#### ACTION 2



Once researchers knew more about COVID-19, responsive measures were put into place to try to stop the spread of the virus as effectively and efficiently as possible. Ideally, effective measures would help slow or even stop the growth rate. Think about the information about growth rates you have learned so far and applied to your mathematical modeling. Now consider how your model might be manipulated to consider the mathematical effects of these measures on growth rate.

Measures like physical distancing and the use of face masks cause the infection to spread more slowly.

- This is expressed by the *effective reproduction number*,  $R_{eff}$ .
- Containment measures (*i.e.*, social distancing, masking) will reduce the  $R$  value.
- If the spread of infection is reduced by  $x\%$ , then the effective reproduction number

would be defined as  $R_{eff} = \left(1 - \frac{x}{100}\right) \cdot R_0$

- This means that if the spread of infection is reduced to 20%, then the effective reproduction number  $R_{eff} = 3.2$ .

**Explore** the GeoGebra applet to find out more about how to contain the spread of COVID-19. With the help of sliders, you can modify the effects of physical distancing and face masks. You can also change the time at which each or both measures are implemented. As you explore, think about how the usefulness of each of these preventive measures is represented numerically. You will find the applet here: <https://www.geogebra.org/m/qavutkx5>.

**Interpret****QUESTION 6**

What does “flattening the curve” mean in mathematical terms?

*Answers will vary. Possible student responses:*

- *My graph in Question 4 showed fast growth (the curve was steep). Preventing COVID-19 cases through measures like social distancing and masking would decrease cases which would impact how steep the curve appears on the graph.*
- *Preventing new cases of COVID-19 reduces the rate of infection which reduces the steepness of the curve in my graph.*

**Evidence of Learning**

- Students will give a written description equating a flat curve to no new COVID-19 cases.

**Assessing/Advancing Questions**

- What does the current graph look like? What is happening?
- How do preventative measures change the curve?

**QUESTION 7**

On the graph in the GeoGebra applet, what does “measures start at” mean?

*Answers will vary. Possible student responses:*

- “**Measures start at**” means *the number of days at which you take a measurement of new COVID-19 cases.*

**Evidence of Learning**

- This is an opportunity to clarify the legend on the graph so students can identify which variables are being changed when they manipulate the sliders on the graph.
- Use the changes in the curve to reference the formula and how it changes.

**Assessing/Advancing Questions**

- How do “**measures start at**” change the graph? What does that mean?

**QUESTION 8**



What do the decimals of “physical distancing effectivity” and “face mask effectivity” mean?

Answers will vary. Possible student responses:

- “**Physical distancing effectivity**” means the percentage of people practicing physical distancing.
- “**Face mask effectivity**” means the percentage of people practicing wearing face masks.

**Evidence of Learning**

- This is an opportunity to clarify the legend on the graph so students can identify which variables are being changed when they manipulate the sliders on the graph.
- Use the changes in the curve to reference the formula and how it changes.

**Assessing/Advancing Questions**

- How does **physical distancing effectivity** change the graph? What does that mean?
- How does **face mask effectivity** change the graph? What does that mean?

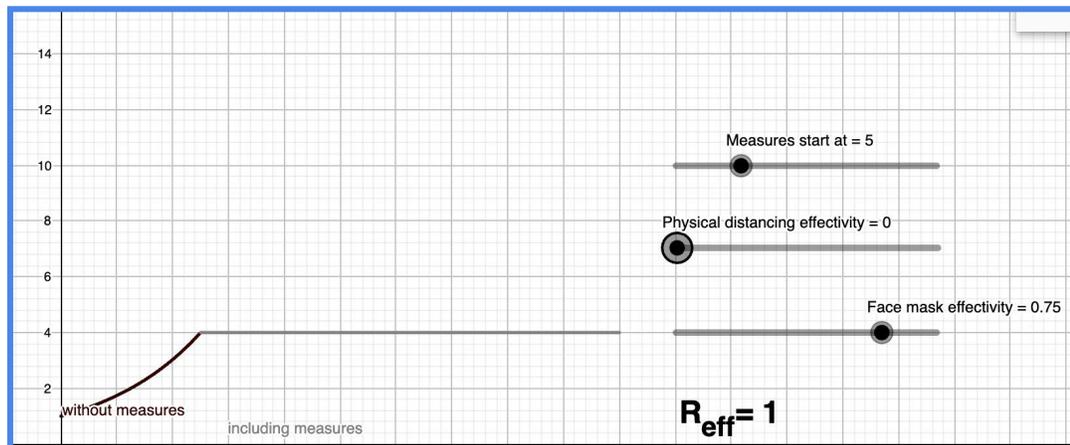
**Validate**

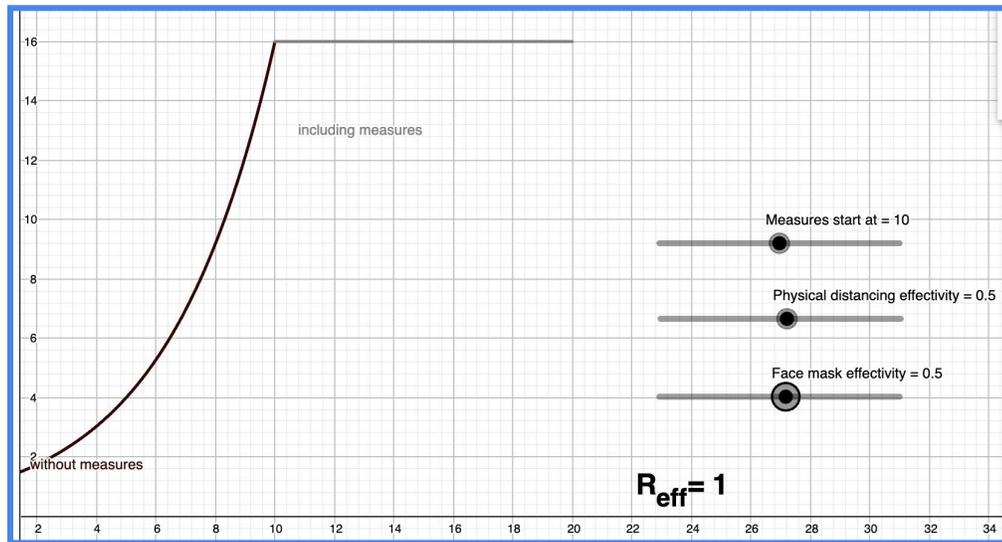
**QUESTION 9**



What combination of masking and social distancing causes the curve to flatten after 5 days, 10 days, and 30 days? How many cases of infection are you preventing?

Answers will vary. Possible student responses:





**Evidence of Learning**

- Students find a combination of physical distancing and face masking that creates a flat line on the graph.
- Students explain that the decimals represent the percentage of people adhering to that preventative measure.

**Assessing/Advancing Questions**

- How much physical distancing do we need to slow the growth of COVID-19? When does the growth stop?
- How much masking do we need to slow the growth of COVID-19? When does the growth stop?

**Manipulate Model and Compute, Interpret, and Validate Checklist**

Requirement	Teacher Notes
Explain the concept of <b>flattening the curve</b> in accurate mathematical terms.	(Teachers, please add your notes here.)
Accurately explain the three different variables from the GeoGebra graph.	(Teachers, please add your notes here.)
Mathematically justify my combination of preventative measures after 5, 10, and 30 days.	(Teachers, please add your notes here.)
Accurately calculate the number of cases that have been prevented.	(Teachers, please add your notes here.)
(Teachers, please add your notes here.)	

## Reflect and Report

In the **Reflect and Report** section, you will use the models you created and manipulated to reflect upon what you learned. You will use what you learned to craft messaging about how best to mitigate the spread of viruses and disease in the future with others. The **Reflect and Report Checklist** can be used to ensure you have the necessary components before finalizing your work on this task.

### Reflect

#### QUESTION 1



What have you learned as a result of this task?

**Considerations:**

- How can studying a pandemic that is almost over lead to modeling and data that will serve in limiting the spread of future diseases?
- What is one efficient, concise message that can come from this study to help those in power better respond to a pandemic next time?

*Answers will vary.*

**Report**

**ACTION 3**



You are a world-renowned immunologist and have been given the role of Chief Medical Officer to the President of the United States.

Given your new role, how would you use your knowledge about the spread of infection, the data you have collected, and the impact of COVID-19 to provide guidance to the president on mitigating future infectious disease and virus outbreaks?

**Write** a concise, clear message that the President can use in a press conference. Make sure your message is:

- At least 1-2 paragraphs;
- Technically sound mathematically and conceptually yet understandable by the average household television audience; and
- Persuasive enough to convince the President and the audience that these measures are statistically sound and absolutely necessary.



Have a classmate try reading your message aloud; help each other with a peer review to refine your messages, making them as concise and clear as possible.

*Answers will vary. Possible student responses might include:*

- *COVID-19 is growing exponentially (one person is infecting 4 people every 5 days).*
- *Data show that prevention measures reduce the exponential growth significantly.*
- *We have data that show physical distancing and masking are effective at flattening the curve when at least half of the population is physical distancing and masking.*

**Evidence of Learning**

- Responses are concise and include data from the task.
- Responses include information about both exponential growth and decay.
- Responses indicate the effectiveness of different prevention measures (physical distancing and masking) on “flattening the curve.”

**Reflect and Report Checklist**

Requirement	Teacher Notes
I have reflected upon what I have learned in this task and how it can be used to help limit the spread of future infectious diseases.	(Teachers, please add your notes here.)
I created a clear, concise message that is appropriate for the identified audience and platform and will be influential in effecting change.	(Teachers, please add your notes here.)
(Teachers, please add your notes here.)	

**Extension**

Students can further engage with the concepts from this task with the extension featured below.

1. Consider the following as it relates to what you have already learned about the mathematics behind the spread of infectious diseases and viruses:
  - a. The percentage of people who are immune (through vaccination or recovery from the virus),  $P_I$  in the total population also affects the  $R_{eff}$  value.
  - b. Taking this effect into account is more complicated because this group becomes larger over time.
  - c. At the beginning of the pandemic,  $P_I$  is zero, whereas at the end, when everybody is immune, the value is 100. The way in which  $P_I$  is used to calculate  $R_{eff}$  is the same as that for physical distancing or face masks: 
$$R_{eff} = \left(1 - \frac{x}{100}\right) \cdot R_0$$
2. Find the minimum value of  $P_I$  required to contain the spread of the virus (*i.e.*, to reduce the R value to 1.)
3. What would happen if, at the start, more than 1 person was infected?
4. Compare the percentage of the population that needs to be immunized to achieve herd immunity for the following illnesses:
  - a. COVID-19 ( $R_0 = 4$ )
  - b. Measles ( $R_0 = 15$ )
  - c. Polio ( $R_0 = 10$ )

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## References

- Crawford, Tom. "Oxford Mathematician explains Exponential Growth for COVID-19 (Coronavirus)." *YouTube*, Tom Rocks Maths, 20 March 2021, <https://www.youtube.com/watch?v=xUVERo2xpH4>.
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